

This presentation contains animations
which require PDF browser which
accepts JavaScript.

For best results use Acrobat Reader.

RSK
oooo

problems
ooo

theorem: bumping routes
ooooo

proof, the easy part
ooooo

proof, science fiction
oooo

the end
ooooo



Robinson-Schensted-Knuth algorithm

Start with two empty tableaux. Read letters of the word one after another. With each letter proceed as follows:

1. start with the bottom row of the insertion tableau P ,
2. insert the letter to the leftmost box in this row which contains a number which is **bigger** than the one which you want to insert,
3. if you had to bump some letter, this bumped letter must be inserted in to the next row according to the rule number 2,
4. if you inserted a letter to an empty box in the insertion tableau P , make a mark about the position of this box in the recording tableau Q and proceed to the next letter of the word.



74	99
23	53
16	41
37	82

insertion tableau $P(w)$

8	9
4	6
1	2
3	5

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, 10)$$

Further reading



Dan Romik,
"The Surprising
Mathematics of Longest
Increasing
Subsequences"
Foreword by Persi Diaconis

legal PDF file available
on author's website

Want more? Visit

→ psniady.impan.pl/surprising

never have seen RSK in your life?
print the handout!

→ psniady.impan.pl/bumping

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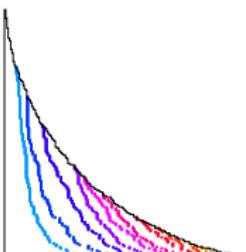
the end
ooooo

Poisson limit of bumping routes in the Robinson–Schensted correspondence

Piotr Śniady

IMPAN Toruń

joint work with Mikołaj Marciniak and Łukasz Maślanka



handout, slides

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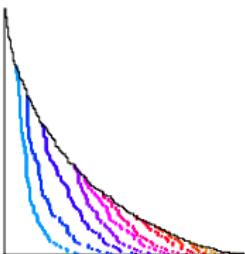
proof, science fiction
oooo

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ooooo

what can you say about RSK with random input?

we apply Robinson–Schensted algorithm
to a very long random sequence;

- what can you say about
bumping routes?
- what is the **trajectory**
of your **favorite number** in the insertion tableau?



Robinson–Schensted–Knuth algorithm is a bijection...

output:

input:

- sequence $w = (w_1, \dots, w_n)$

- semistandard tableau P ,
- standard tableau Q ,

P and Q have the same shape
with n boxes

example:

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41)$$

74	99		
23	53	70	
16	37	41	82

insertion tableau $P(w)$

8	9		
4	6	7	
1	2	3	5

recording tableau $Q(w)$

RSK
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Robinson–Schensted–Knuth algorithm — the induction step

74	99		
23	53	70	
16	37	41	82

insertion tableau $P(w)$

8	9		
4	6	7	
1	2	3	5

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41)$$

RSK
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Robinson–Schensted–Knuth algorithm — the induction step

74	99		
23	53	70	
16	37	41	82

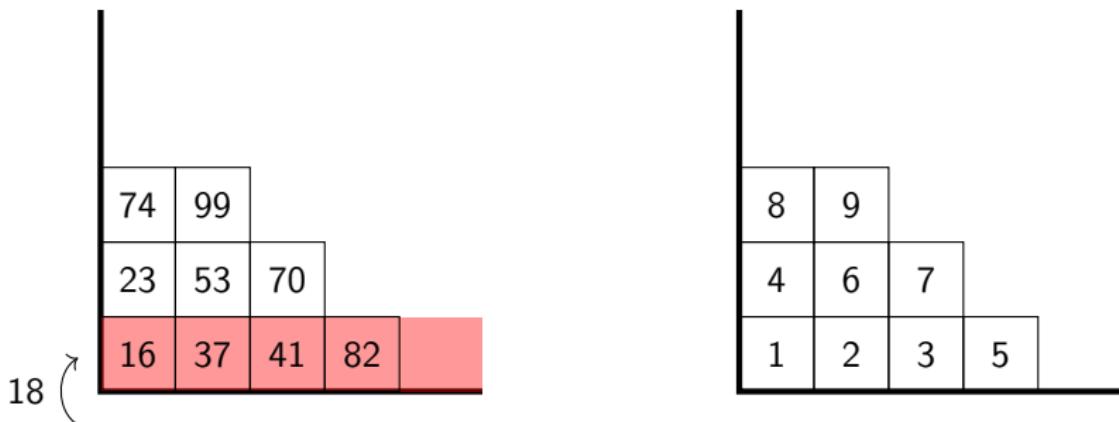
insertion tableau $P(w)$

8	9		
4	6	7	
1	2	3	5

recording tableau $Q(w)$

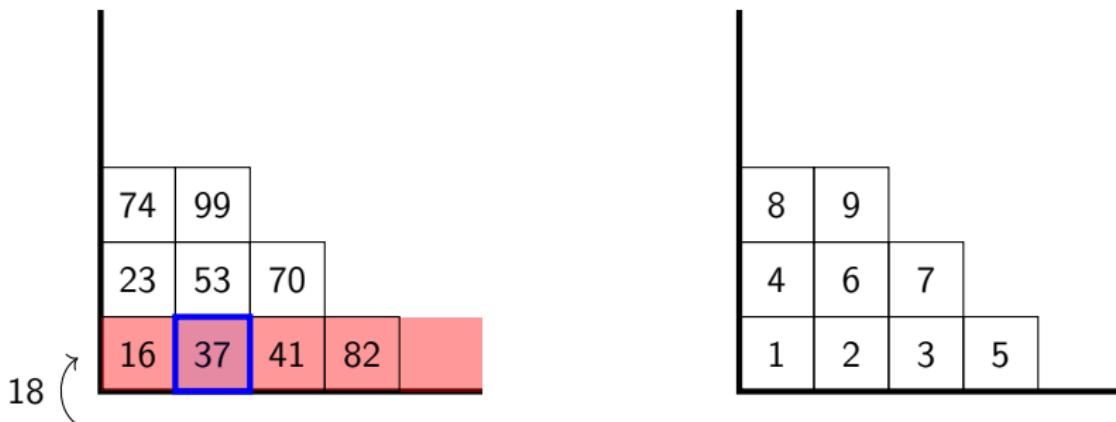
$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

Robinson–Schensted–Knuth algorithm — the induction step

insertion tableau $P(w)$ recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

Robinson–Schensted–Knuth algorithm — the induction step

insertion tableau $P(w)$ recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

Robinson–Schensted–Knuth algorithm — the induction step

74	99						
23	53	70					
16	37	41	82	37	41	18	

insertion tableau $P(w)$

8	9		
4	6	7	
1	2	3	5

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, 18)$$

Robinson–Schensted–Knuth algorithm — the induction step

74	99			
23	53	70		
16	41	82		

insertion tableau $P(w)$

8	9			
4	6	7		
1	2	3	5	

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

Robinson–Schensted–Knuth algorithm — the induction step



74	99			
23	53	70		
16	18	41	82	

insertion tableau $P(w)$

8	9			
4	6	7		
1	2	3	5	

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

Robinson–Schensted–Knuth algorithm — the induction step

74	99			
23	53	70		
16	18	41	82	

insertion tableau $P(w)$

8	9		
4	6	7	
1	2	3	5

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

Robinson–Schensted–Knuth algorithm — the induction step

74	99		
23	53	70	
16	18	41	82

insertion tableau $P(w)$

8	9		
4	6	7	
1	2	3	5

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

Robinson–Schensted–Knuth algorithm — the induction step

74	99			
23	53	70		
16	18	41	82	
37				

insertion tableau $P(w)$

8	9		
4	6	7	
1	2	3	5

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

Robinson–Schensted–Knuth algorithm — the induction step

74	99			
23			70	
16	18	41	82	

53
37insertion tableau $P(w)$

8	9			
4	6	7		
1	2	3	5	

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

Robinson–Schensted–Knuth algorithm — the induction step

74	99		
23	37	70	
16	18	41	82

53 ↗

8	9		
4	6	7	
1	2	3	5

insertion tableau $P(w)$ recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

Robinson–Schensted–Knuth algorithm — the induction step

A 3x4 grid of boxes representing the insertion tableau $P(w)$. The entries are:

74	99		
23	37	70	
16	18	41	82

The number 53 is circled on the left side of the first row.

insertion tableau $P(w)$

A 4x5 grid of boxes representing the recording tableau $Q(w)$. The entries are:

8	9			
4	6	7		
1	2	3	5	

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

Robinson–Schensted–Knuth algorithm — the induction step

The diagram shows a 3x4 grid of numbers. The first two columns are highlighted in blue, and the last two columns are highlighted in red. A circled number 53 is placed to the left of the first column, with an arrow pointing towards it.

74	99		
23	37	70	
16	18	41	82

insertion tableau $P(w)$

The diagram shows a 4x5 grid of numbers. The first three columns are highlighted in blue, and the last two columns are highlighted in red. The numbers are arranged in a staircase pattern.

8	9			
4	6	7		
1	2	3	5	

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

RSK
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○○○theorem: bumping routes
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○○○○the end
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Robinson–Schensted–Knuth algorithm — the induction step

74	99		
53	23	37	70
	16	18	41
			82

insertion tableau $P(w)$

8	9		
4	6	7	
1	2	3	5

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

Robinson–Schensted–Knuth algorithm — the induction step

74				
53				
23	37	70		
16	18	41	82	

insertion tableau $P(w)$

8	9			
4	6	7		
1	2	3	5	

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

Robinson–Schensted–Knuth algorithm — the induction step

insertion tableau $P(w)$

53	99					
23	37	70				
16	18	41	82			

recording tableau $Q(w)$

8	9		
4	6	7	
1	2	3	5

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

Robinson–Schensted–Knuth algorithm — the induction step

53	99		
23	37	70	
16	18	41	82

insertion tableau $P(w)$

8	9		
4	6	7	
1	2	3	5

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

Robinson–Schensted–Knuth algorithm — the induction step

53	99								
23	37	70							
16	18	41	82						

insertion tableau $P(w)$

8	9								
4	6	7							
1	2	3	5						

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

Robinson–Schensted–Knuth algorithm — the induction step

74				
53	99			
23	37		70	
16	18	41	82	

insertion tableau $P(w)$

8	9		
4	6	7	
1	2	3	5

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

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Robinson–Schensted–Knuth algorithm — the induction step

74			
53	99		
23	37	70	
16	18	41	82

insertion tableau $P(w)$

8	9		
4	6	7	
1	2	3	5

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

Robinson–Schensted–Knuth algorithm — the induction step

74			
53	99		
23	37	70	
16	18	41	82

insertion tableau $P(w)$

10		
8	9	
4	6	7
1	2	3
		5

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

Robinson–Schensted–Knuth algorithm — the induction step

bumping route

74			
53	99		
23	37	70	
16	18	41	82

new box

10			
8	9		
4	6	7	
1	2	3	5

insertion tableau $P(w)$ recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

RSK
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Robinson–Schensted–Knuth algorithm — the induction step

74			
53	99		
23	37	70	
16	18	41	82

insertion tableau $P(w)$

10			
8	9		
4	6	7	
1	2	3	5

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, 18)$$

RSK
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problems
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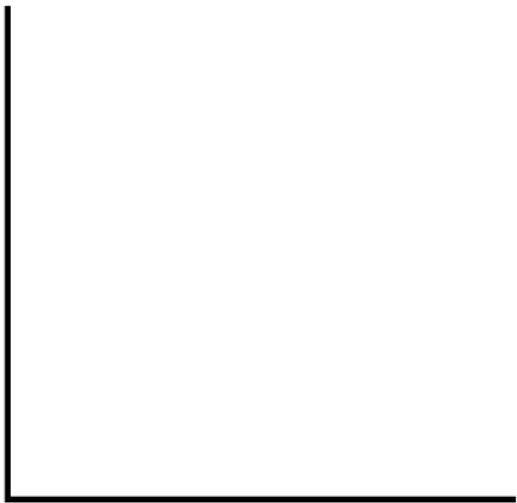
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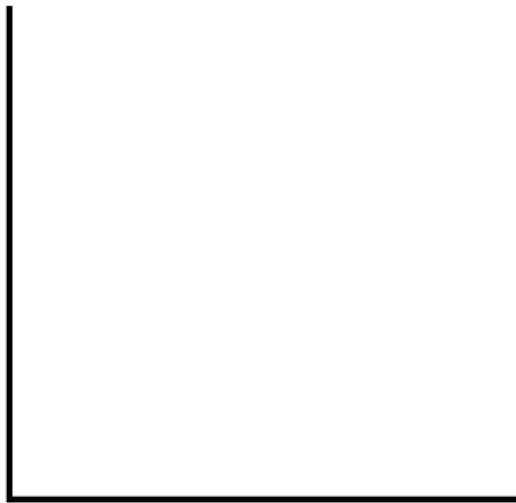
proof, science fiction
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the end
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Robinson–Schensted–Knuth algorithm



insertion tableau $P(w)$



recording tableau $Q(w)$

$$w = \emptyset$$

Robinson–Schensted–Knuth algorithm

23

insertion tableau $P(w)$

1

recording tableau $Q(w)$

$$w = (23)$$

RSK
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problems
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theorem: bumping routes
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proof, the easy part
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proof, science fiction
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the end
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Robinson–Schensted–Knuth algorithm

23	53
----	----

insertion tableau $P(w)$

1	2
---	---

recording tableau $Q(w)$

$$w = (23, \textcolor{blue}{53})$$

Robinson–Schensted–Knuth algorithm

23	53	74

insertion tableau $P(w)$

1	2	3

recording tableau $Q(w)$

$$w = (23, 53, 74)$$

RSK
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the end
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Robinson–Schensted–Knuth algorithm

23		
16	53	74

insertion tableau $P(w)$

4		
1	2	3

recording tableau $Q(w)$

$$w = (23, 53, 74, \textcolor{blue}{16})$$

RSK
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problems
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theorem: bumping routes
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proof, the easy part
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proof, science fiction
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the end
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Robinson–Schensted–Knuth algorithm

23				
16	53	74	99	

insertion tableau $P(w)$

4				
1	2	3	5	

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99)$$

Robinson–Schensted–Knuth algorithm

23	74
16	53
70	99

insertion tableau $P(w)$

4	6
1	2
3	5

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70)$$

Robinson–Schensted–Knuth algorithm

23	74	99	
16	53	70	82

insertion tableau $P(w)$

4	6	7	
1	2	3	5

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, \textcolor{green}{99}, 70, \textcolor{blue}{82})$$

Robinson–Schensted–Knuth algorithm

74				
23	53	99		
16	37	70	82	

insertion tableau $P(w)$

8				
4	6	7		
1	2	3	5	

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37)$$

Robinson–Schensted–Knuth algorithm

74	99		
23	53	70	
16	37	41	82

insertion tableau $P(w)$

8	9		
4	6	7	
1	2	3	5

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, \textcolor{green}{99}, 70, 82, 37, \textcolor{blue}{41})$$

Robinson–Schensted–Knuth algorithm

74			
53	99		
23	37	70	
16	34	41	82

insertion tableau $P(w)$

10			
8	9		
4	6	7	
1	2	3	5

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, 34)$$

Robinson–Schensted–Knuth algorithm

74				
53	99			
23	37	70	82	
16	34	41	73	

insertion tableau $P(w)$

10				
8	9			
4	6	7	11	
1	2	3	5	

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, \textcolor{green}{99}, 70, 82, 37, 41, 34, \textcolor{blue}{73})$$

Robinson–Schensted–Knuth algorithm

74			
53			
23	99		
16	37	70	82
2	34	41	73

insertion tableau $P(w)$

12			
10			
8	9		
4	6	7	11
1	2	3	5

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, \textcolor{red}{99}, 70, 82, 37, 41, 34, 73, \textcolor{blue}{2})$$

Robinson–Schensted–Knuth algorithm

74				
53	99			
23	37			
16	34	70	82	
2	24	41	73	

insertion tableau $P(w)$

12				
10	13			
8	9			
4	6	7	11	
1	2	3	5	

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, \textcolor{red}{99}, 70, 82, 37, 41, 34, 73, 2, \textcolor{red}{24})$$

why RSK?

- understanding irreducible representations of the symmetric groups,
- tool for Littlewood–Richardson coefficients,
- RSK applied to random inputs (of various types) produces lots of interesting random walks, famous random Young diagrams and Young tableaux,
- amazing bijection with lots of magic symmetries,

magic symmetry:

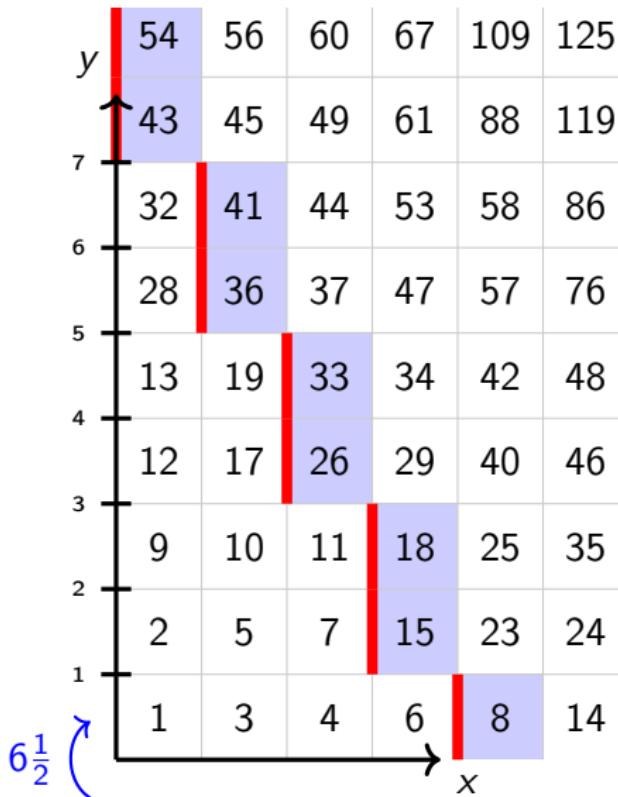
if w_1, \dots, w_n are all different, then

$$P(\underbrace{w_n, \dots, w_1}_{\text{the word } w \text{ read backwards}}) = \left[P(w_1, \dots, w_n) \right]^{\text{transpose}}$$

let ξ_1, ξ_2, \dots
be independent random variables
with the uniform distribution
on the unit interval $[0, 1]$

what can you say about
the infinite bumping route

$$Q(\xi_1, \xi_2, \dots) \leftarrow m + \frac{1}{2} ?$$



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let ξ_1, ξ_2, \dots be independent random variables
with the uniform distribution on the unit interval $[0, 1]$

where is your favourite number ∞ in the insertion tableau

$$P(\xi_1, \dots, \xi_m, \infty, \xi_{m+1}, \dots, \xi_t)?$$

bumping route = trajectory

let $Q = Q(\xi_1, \xi_2, \dots)$ be a (finite or infinite) recording tableau;
then

the bumping route related to the insertion $Q \leftarrow m + 1/2$

is equal to

the trajectory of ∞ in the sequence of insertions

$$P(\xi_1, \dots, \xi_m, \infty) \leftarrow \xi_{m+1} \leftarrow \xi_{m+2} \leftarrow \dots$$

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important probability distributions

$\text{Exp}(r)$ is the **exponential distribution** with parameter $r > 0$

'time of waiting until the bus arrives'

$$\mathbb{E} \text{Exp}(r) = \frac{1}{r}$$

$$\text{Erlang}(4) = \text{Exp}(1) + \text{Exp}(1) + \text{Exp}(1) + \text{Exp}(1)$$

'time of waiting until the fourth bus arrives'

RSK
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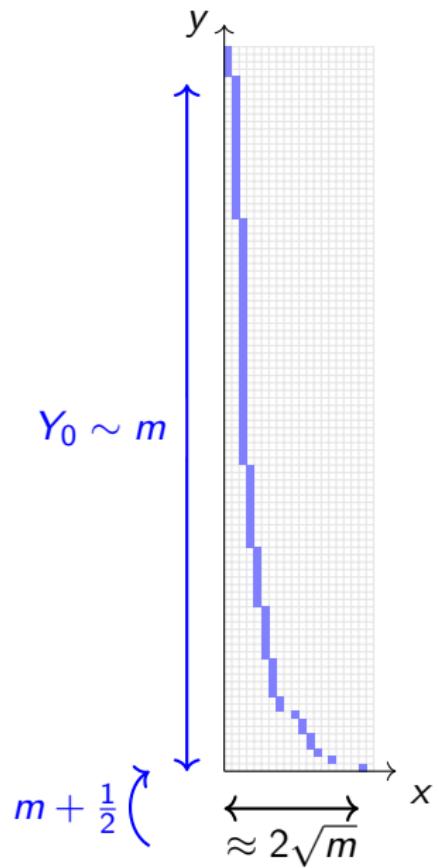
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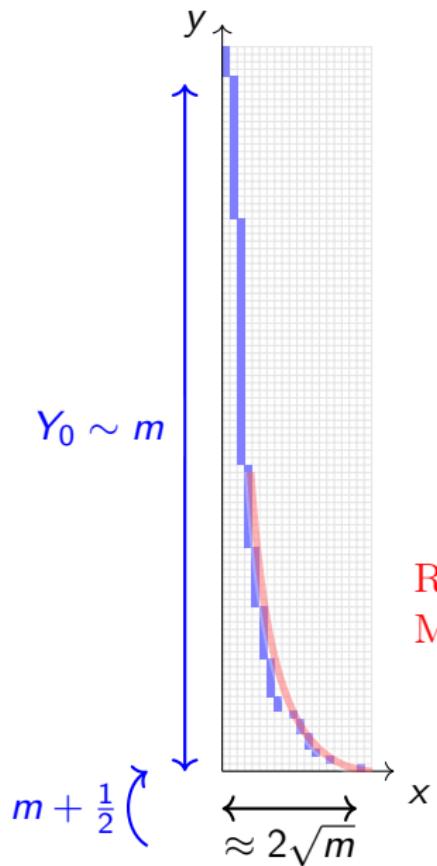
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MARCINIAK 2020

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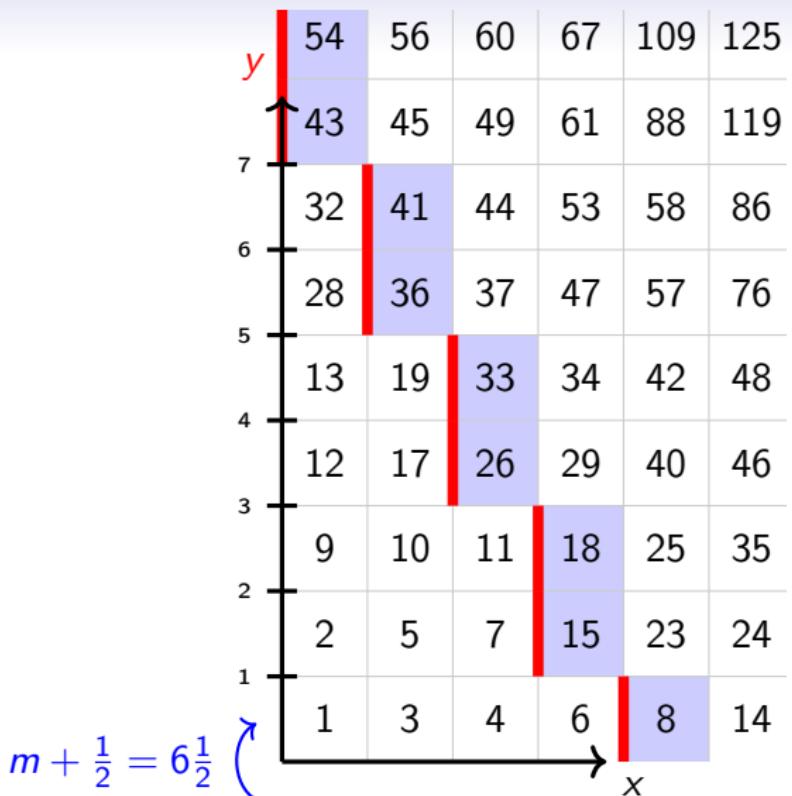
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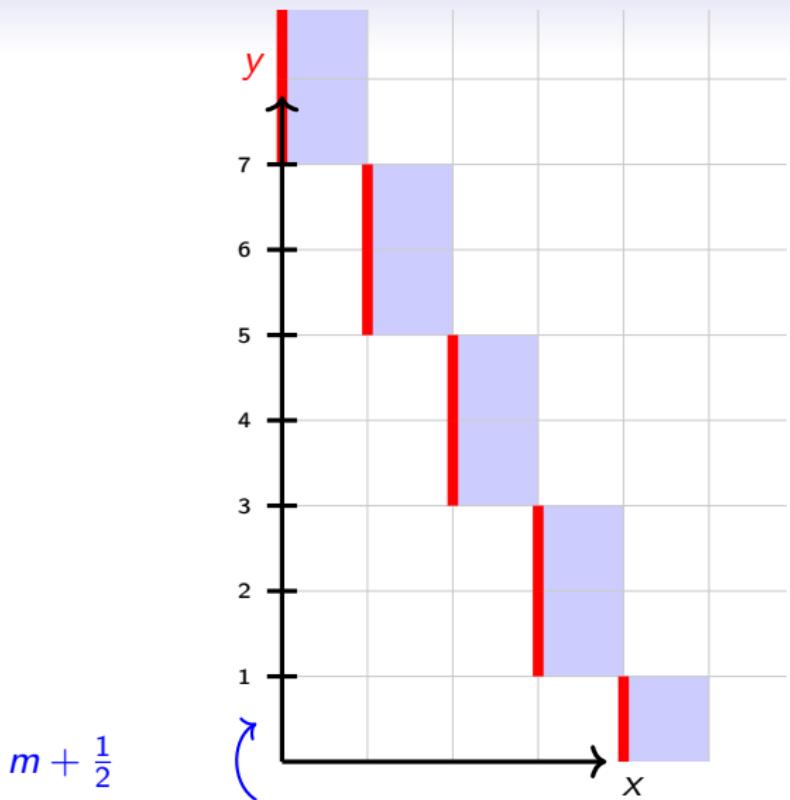
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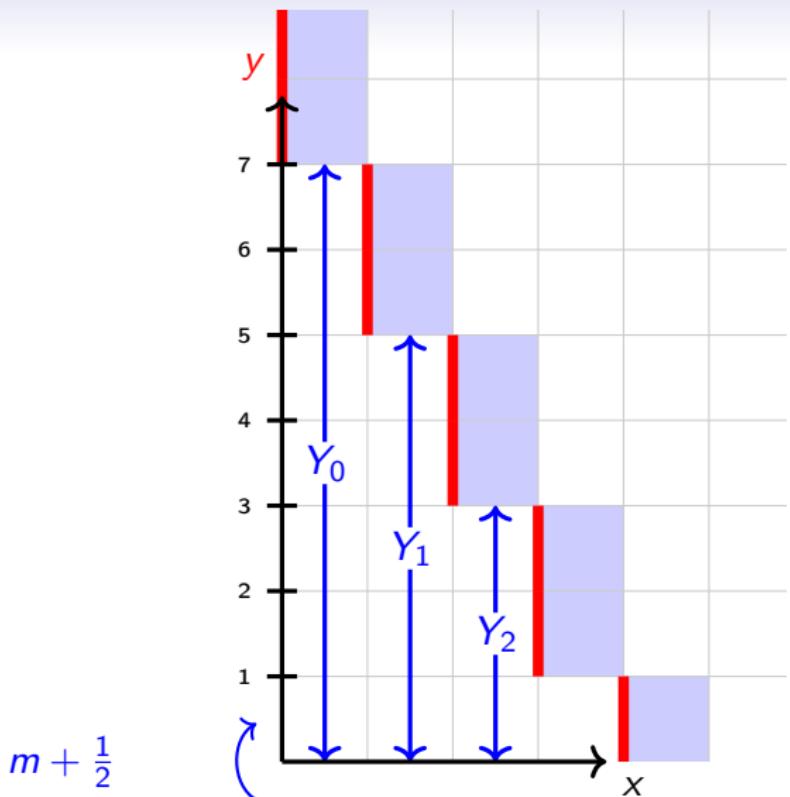
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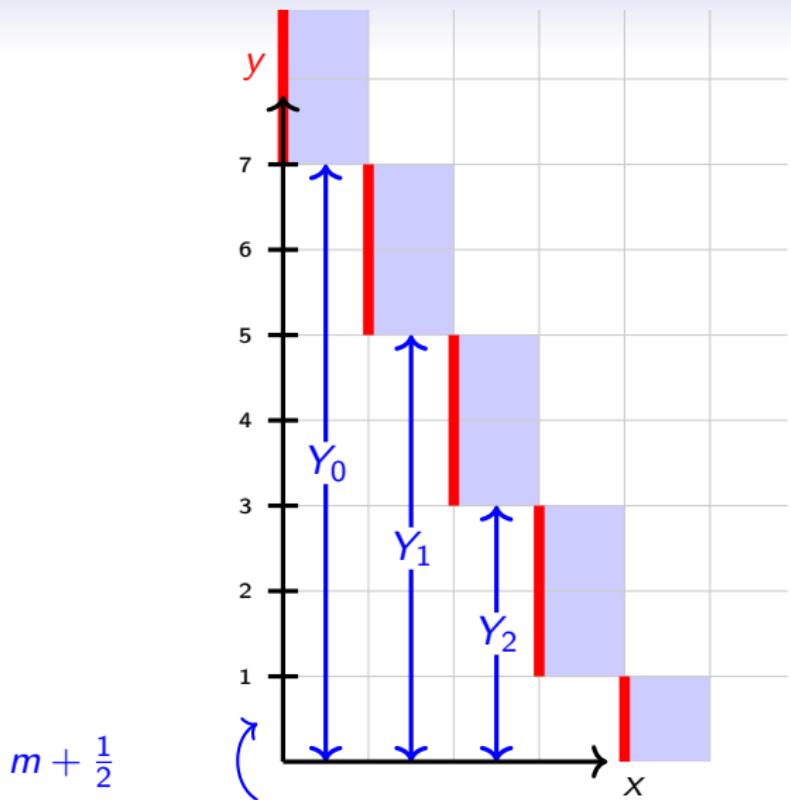
problems
○○○

theorem: bumping routes
○○●○○

proof, the easy part
○○○○○

proof, science fiction
○○○○

the end
○○○○○



for each $m \geq 1$ $\mathbb{P}(Y_0 < \infty) = 1$

RSK
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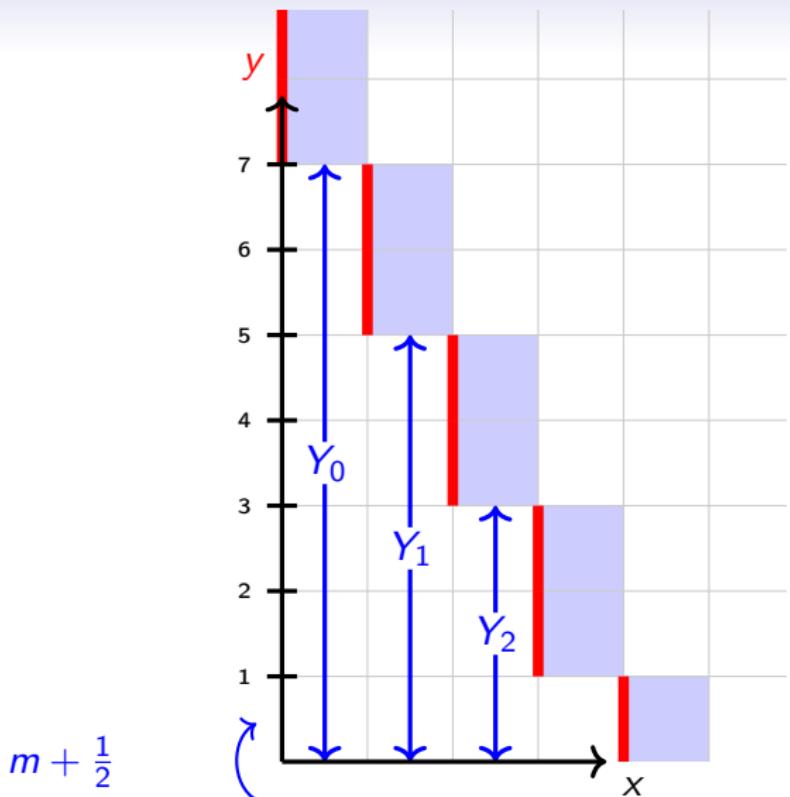
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proof, science fiction
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the end
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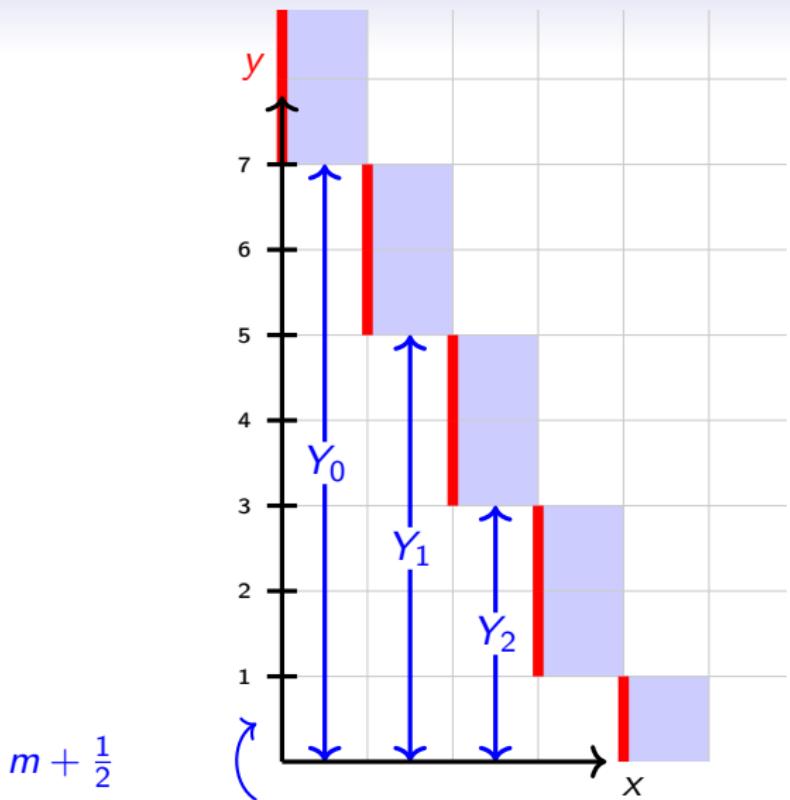
problems
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theorem: bumping routes
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proof, the easy part
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proof, science fiction
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the end
ooooo



for each $m \geq 1$ $\mathbb{E} Y_0 = \infty$

RSK
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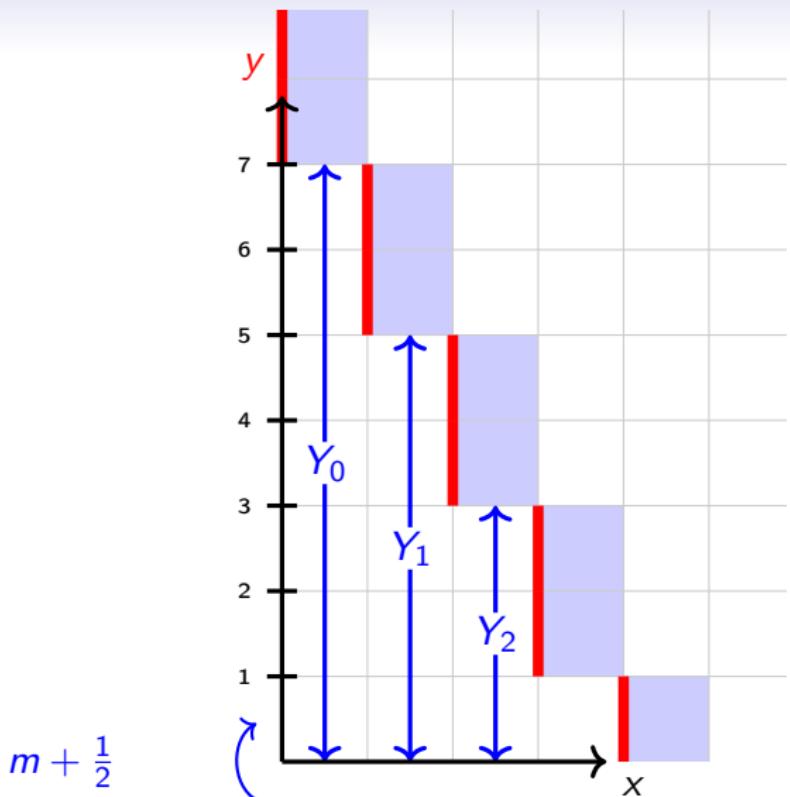
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proof, the easy part
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the end
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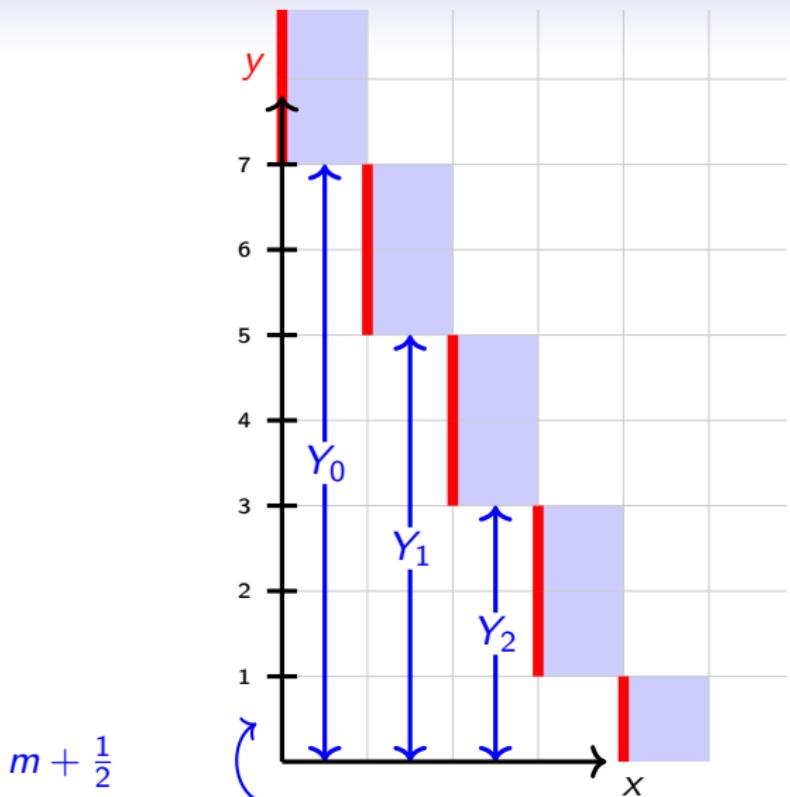
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proof, science fiction
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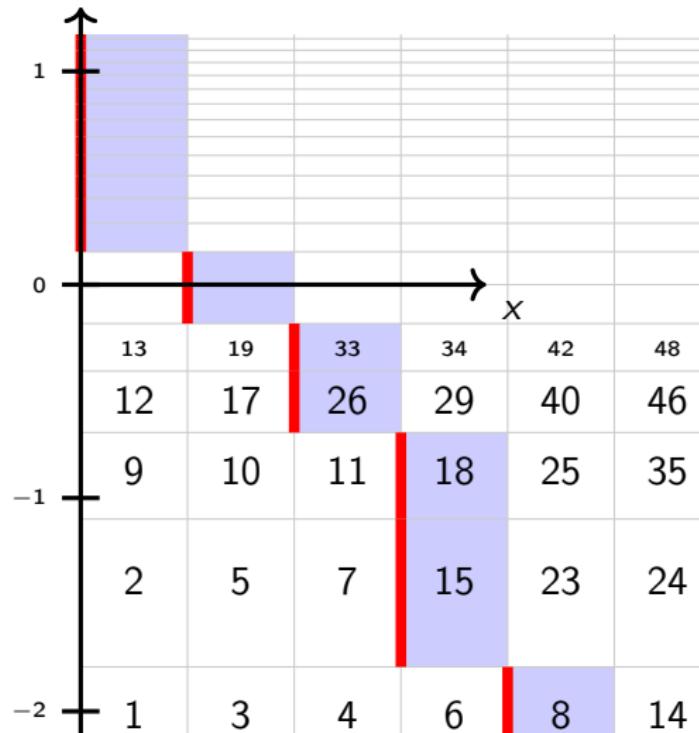
the end
○○○○○



$$\left(\frac{Y_0}{m}, \frac{Y_1}{m}, \dots, \frac{Y_3}{m} \right) \xrightarrow[m \rightarrow \infty]{\text{dist}} ?$$

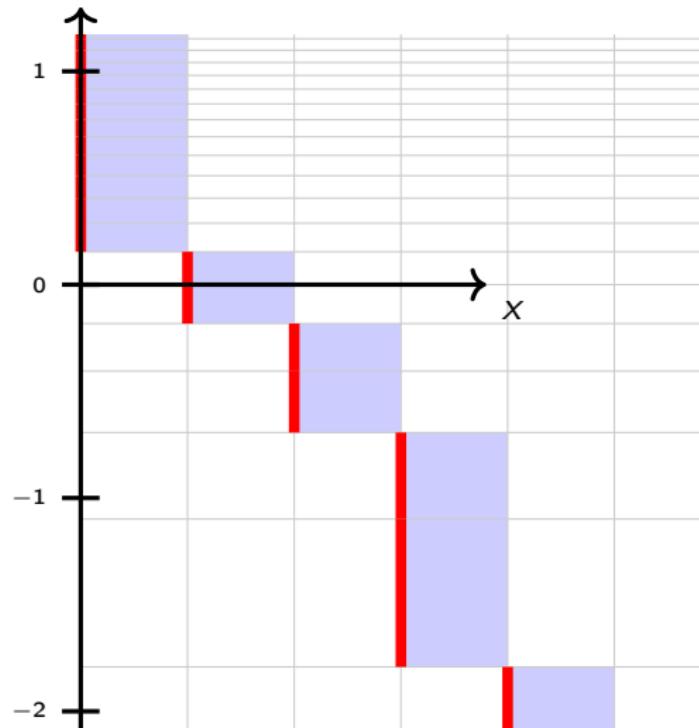
semi-logarithmic plot,

$\log \frac{y}{m}$

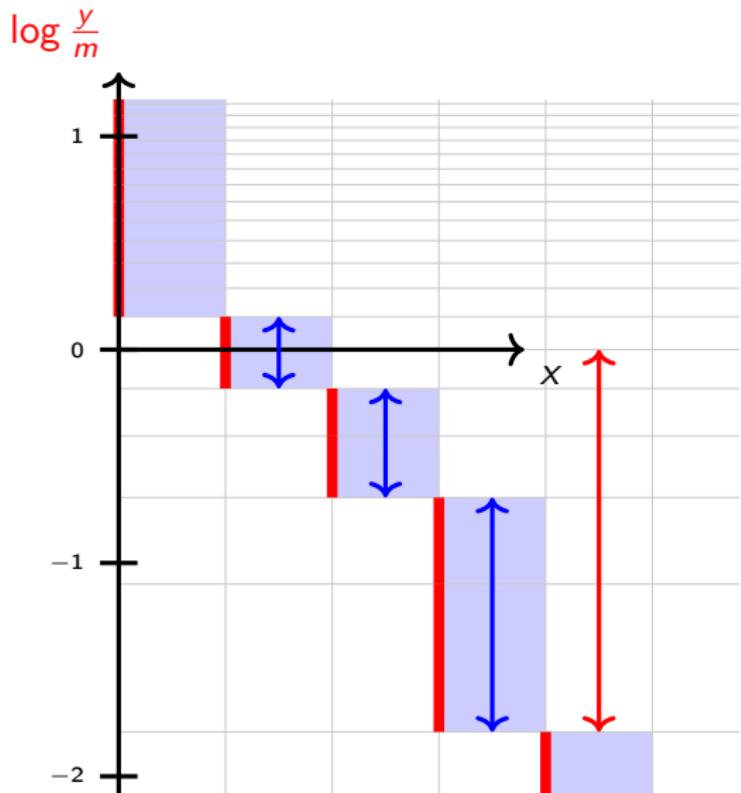


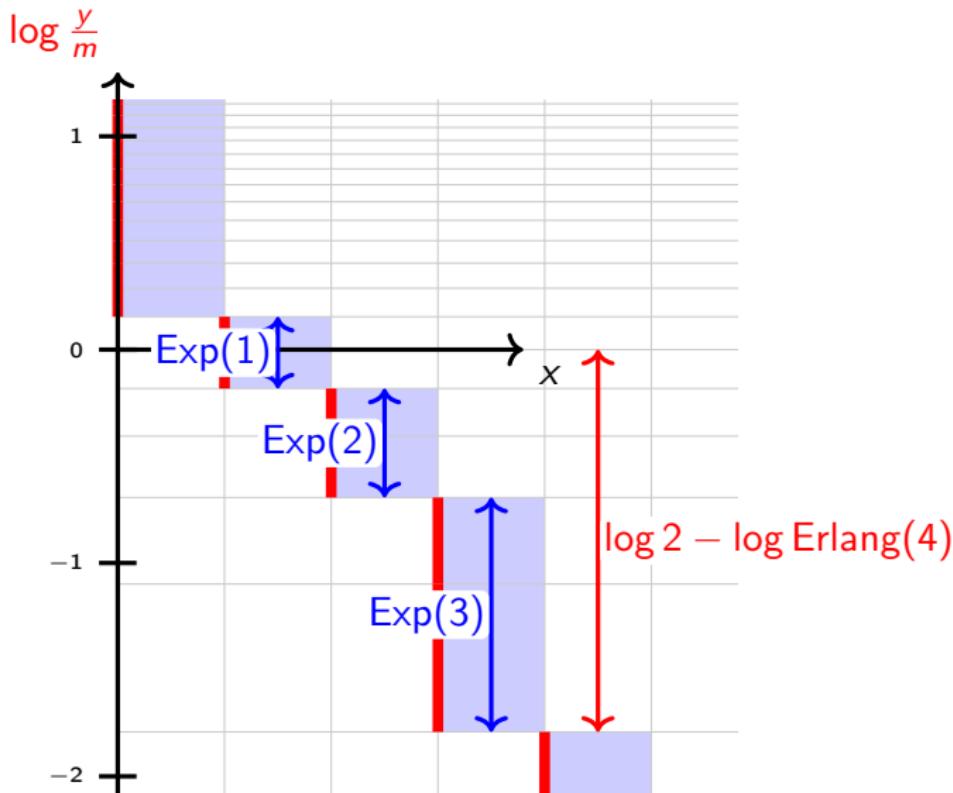
semi-logarithmic plot,

$\log \frac{y}{m}$

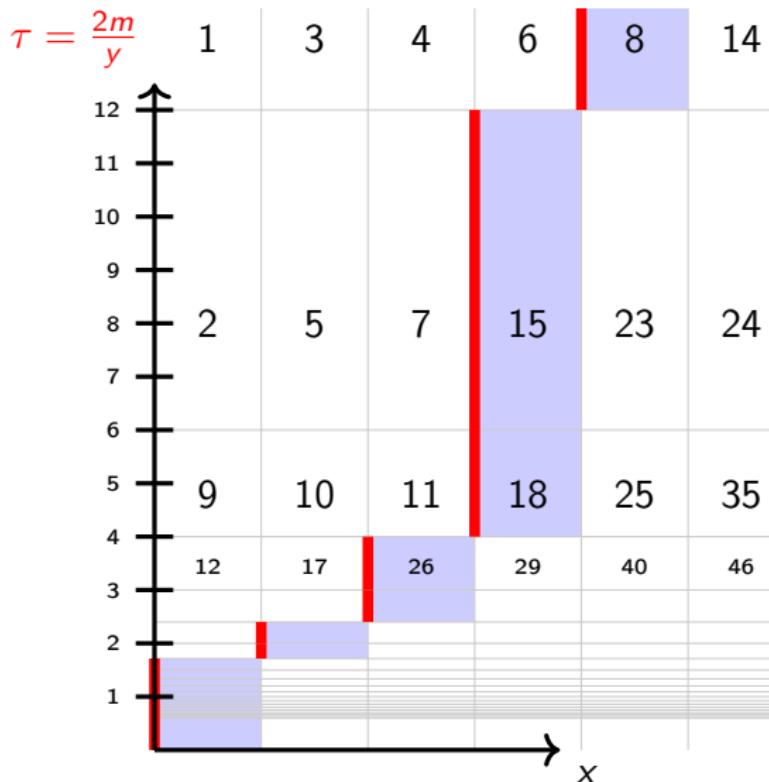


semi-logarithmic plot,



semi-logarithmic plot, $m \rightarrow \infty$ 

semi-projective plot,



RSK
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problems
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theorem: bumping routes
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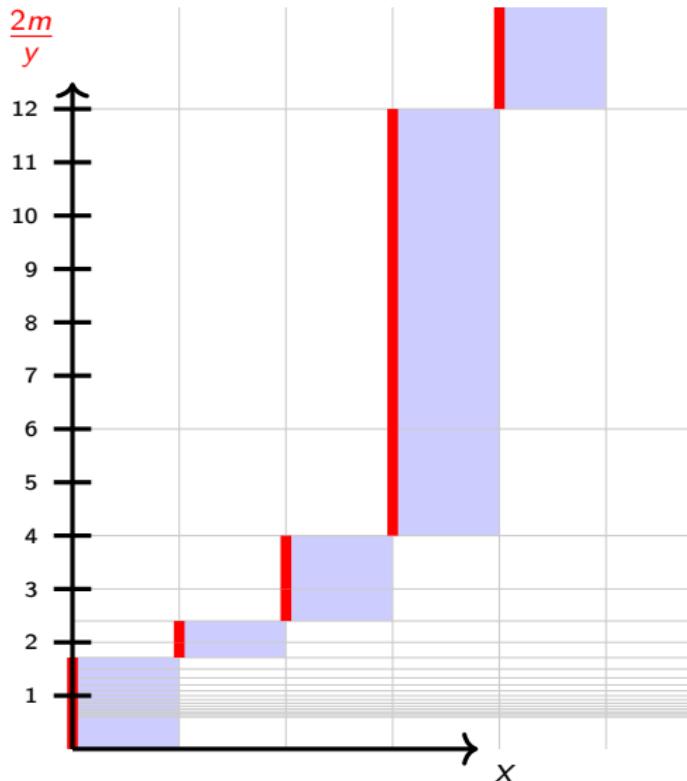
proof, the easy part
ooooo

proof, science fiction
oooo

the end
ooooo

semi-projective plot,

$$\tau = \frac{2m}{y}$$



RSK
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problems
ooo

theorem: bumping routes
oooo●

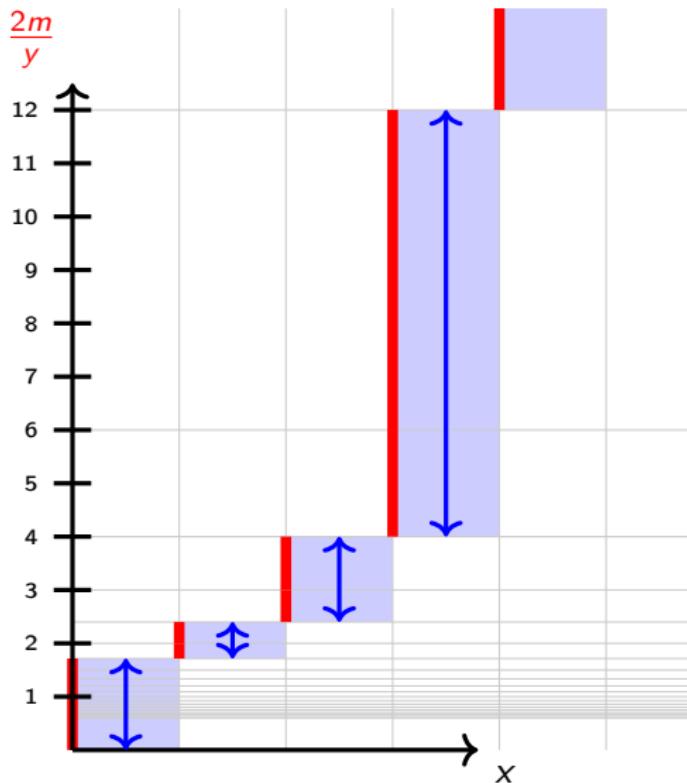
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proof, science fiction
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the end
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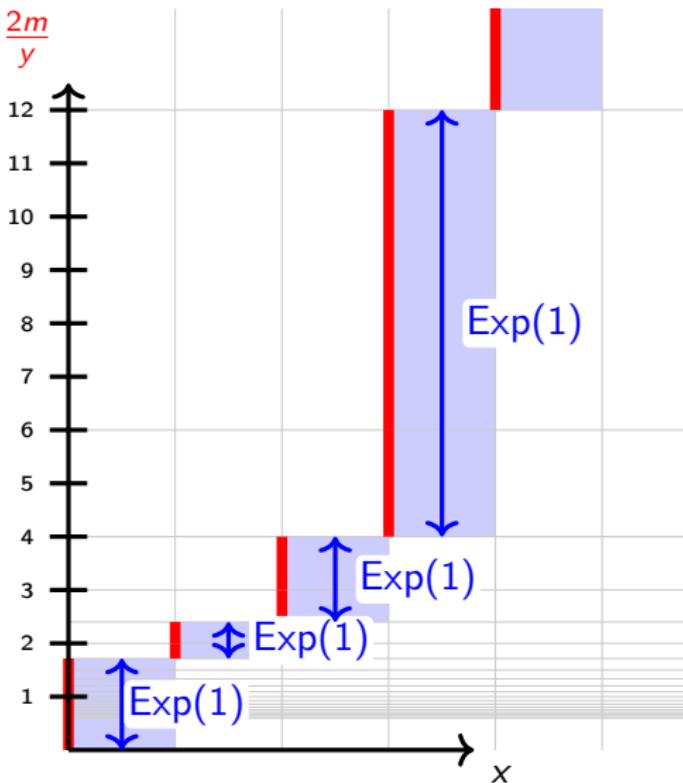
semi-projective plot,

$$\tau = \frac{2m}{y}$$



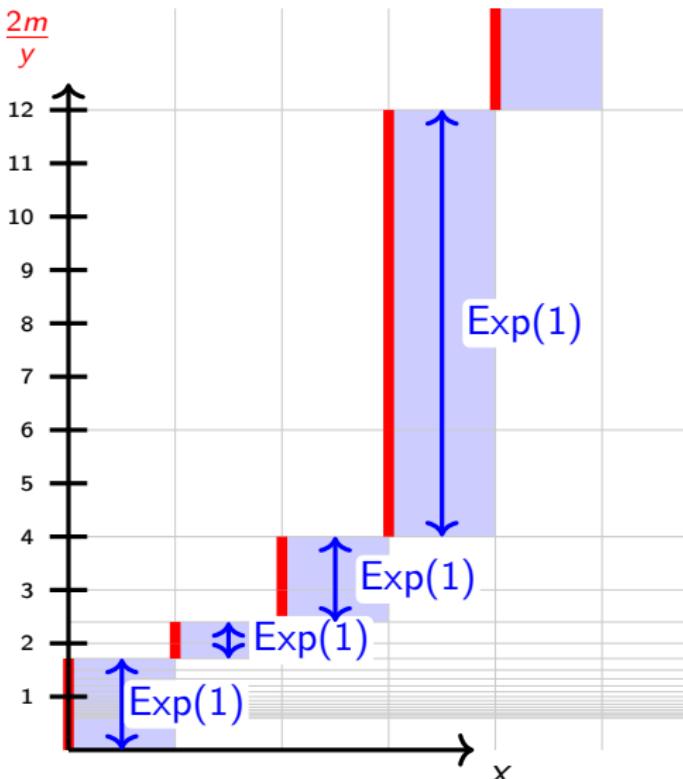
semi-projective plot, $m \rightarrow \infty$

$$\tau = \frac{2m}{y}$$



semi-projective plot, $m \rightarrow \infty$

$$\tau = \frac{2m}{y}$$



Corollary: the red line is a plot of the standard Poisson process

RSK
oooo

problems
ooo

theorem: bumping routes
ooooo

proof, the easy part
●oooo

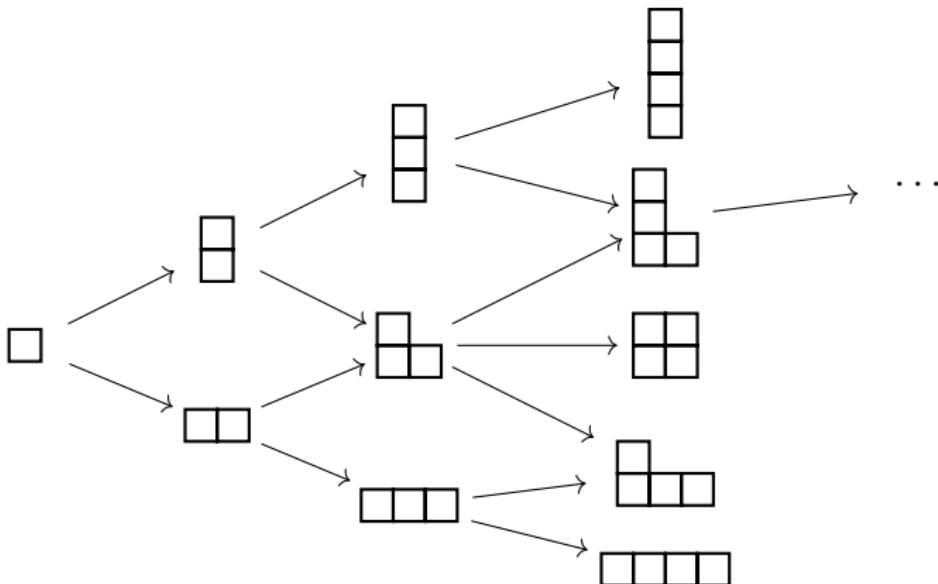
proof, science fiction
oooo

the end
ooooo

what do you see in an insertion tableau
if you ignore the entries?

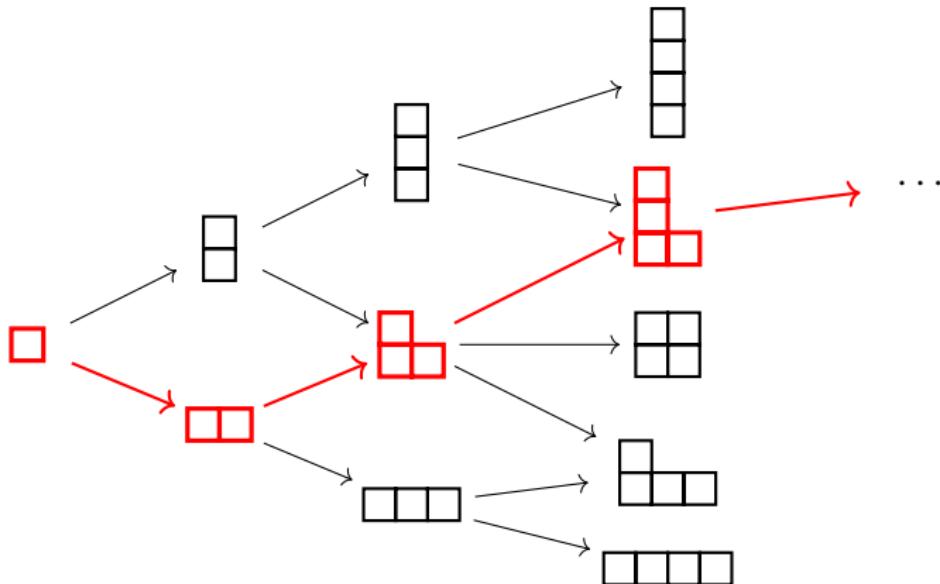
$$\text{shape} \left(\begin{array}{|c|c|} \hline 4 & 8 \\ \hline 3 & 7 & 9 \\ \hline \end{array} \right) = \begin{array}{|c|c|c|} \hline & & \\ \hline & & \\ \hline & & \\ \hline \end{array}$$

what do you see in an insertion tableau
if you ignore the entries?



Plancherel growth process

$$\lambda^{(1)} \nearrow \lambda^{(2)} \nearrow \dots$$



define $\lambda^{(t)} = \text{shape } P(\xi_1, \dots, \xi_t)$ to be the shape
of the insertion tableau related to the prefix of ξ

RSK
○○○○problems
○○○theorem: bumping routes
○○○○○proof, the easy part
○○●○○proof, science fiction
○○○○the end
○○○○○

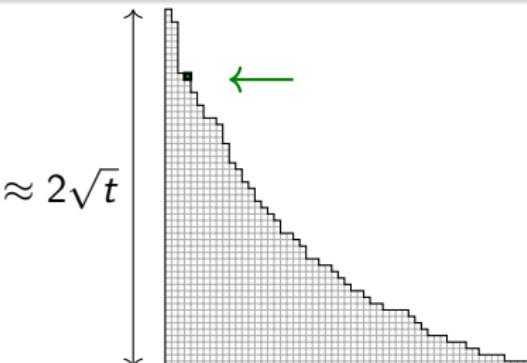
let $m = O(\sqrt{t})$

let (x_t, y_t) be the coordinates of ∞ in the insertion tableau

$$P(\underbrace{\xi_1, \dots, \xi_m}_m, \infty, \xi_{m+1}, \dots, \xi_t)$$

$$y_t \approx 2\sqrt{t},$$

$$x_t = ?$$



RSK
○○○○problems
○○○theorem: bumping routes
○○○○○proof, the easy part
○○○●○proof, science fiction
○○○○the end
○○○○○

let $m = O(\sqrt{n})$

let (x, y) be the coordinates of ∞ in the insertion tableau

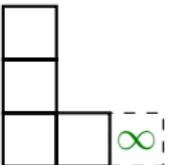
$$P\left(\underbrace{\xi_1, \dots, \xi_n}_n, \infty, \underbrace{\xi_{n+1}, \dots, \xi_{n+m}}_m\right)$$

then

$$y \stackrel{\text{dist}}{\approx} \text{Pois}\left(\frac{m}{\sqrt{n}}\right)$$

this is a result about bottom rows in Plancherel growth process

→ improved version of a result of ALDOUS and DIACONIS



RSK
○○○○problems
○○○theorem: bumping routes
○○○○○proof, the easy part
○○○●○proof, science fiction
○○○○the end
○○○○○

let $m = O(\sqrt{n})$

let (x, y) be the coordinates of ∞ in the insertion tableau

$$P\left(\underbrace{\xi_1, \dots, \xi_n}_n, \infty, \underbrace{\xi_{n+1}, \dots, \xi_{n+m}}_m\right)$$

then

$$y \stackrel{\text{dist}}{\approx} \text{Pois}\left(\frac{m}{\sqrt{n}}\right)$$

RSK
○○○○problems
○○○theorem: bumping routes
○○○○○proof, the easy part
○○○●○proof, science fiction
○○○○the end
○○○○○

let $m = O(\sqrt{n})$

let (x, y) be the coordinates of ∞ in the insertion tableau

$$P\left(\underbrace{\xi_1, \dots, \xi_n}_{n}, \infty, \underbrace{\xi_{n+1}, \dots, \xi_{n+m}}_{m}\right)$$

then

$$y \stackrel{\text{dist}}{\approx} \text{Pois}\left(\frac{m}{\sqrt{n}}\right)$$

Hint: read the word backwards? RSK gives the transpose!

RSK
ooooproblems
oootheorem: bumping routes
oooooproof, the easy part
oooo●proof, science fiction
oooothe end
ooooo

let $m = O(\sqrt{n})$

let (x, y) be the coordinates of ∞ in the insertion tableau

$$P\left(\underbrace{\xi_1, \dots, \xi_m}_m, \infty, \underbrace{\xi_{n+1}, \dots, \xi_{n+m}}_n\right)$$

then

$$\textcolor{red}{x} \stackrel{\text{dist}}{\approx} \text{Pois}\left(\frac{m}{\sqrt{n}}\right)$$

Hint: read the word backwards? RSK gives the transpose!

RSK
○○○○problems
○○○theorem: bumping routes
○○○○○proof, the easy part
○○○○○proof, science fiction
●○○○the end
○○○○○

what you do see in an insertion tableau
if you ignore the entries, except for ∞ ?

$$\text{shape} \left(\begin{array}{|c|c|c|} \hline 4 & \infty & \\ \hline 3 & 7 & 9 \\ \hline \end{array} \right) = \begin{array}{|c|c|c|} \hline & & \infty \\ \hline & & \\ \hline & & \\ \hline \end{array}$$

augmented Plancherel growth process

$$\Lambda^{(t)} = \text{shape } P(\xi_1, \dots, \xi_m, \infty, \xi_{m+1}, \dots, \xi_t)$$

is a Markov chain

RSK
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problems
ooo

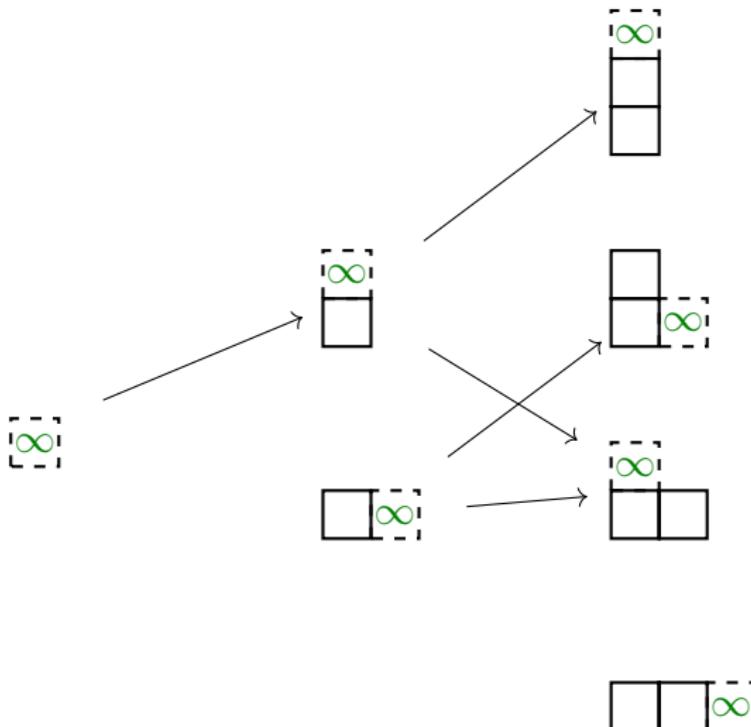
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o●oo

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RSK
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○○○theorem: bumping routes
○○○○○proof, the easy part
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○○●○the end
○○○○○

component 1: probability distribution of $\Lambda^{(t)}$

$$\begin{array}{|c|c|c|}\hline \square & \square & \square \\ \hline \square & \square & \square \\ \hline \end{array} \quad \text{x-coordinate of } \infty = \left(\text{x-coordinate of } \infty, \begin{array}{|c|c|c|}\hline \square & \square & \square \\ \hline \end{array} \right)$$

$$\Lambda^{(t)} \stackrel{\text{dist}}{\approx} \text{Pois} \left(\frac{m}{\sqrt{t}} \right) \times \text{Plancherel}(t)$$

bad news: the result of ALDOUS and DIACONIS is not enough

RSK
○○○○problems
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○○○○○proof, science fiction
○○○●the end
○○○○○

component 2: transition probabilities

suppose that Markov chain Λ at time t
has probability distribution

$$\Lambda^{(t)} \stackrel{\text{dist}}{\approx} \text{Pois}\left(\frac{m}{\sqrt{t}}\right) \times \text{Plancherel}(t)$$

then for $u > t$

$$\Lambda^{(u)} \stackrel{\text{dist}}{\approx} \text{Pois}\left(\frac{m}{\sqrt{u}}\right) \times \text{Plancherel}(u)$$

RSK
○○○problems
○○○theorem: bumping routes
○○○○○proof, the easy part
○○○○○proof, science fiction
○○○●the end
○○○○○

component 2: transition probabilities

suppose that Markov chain Λ at time t
has probability distribution

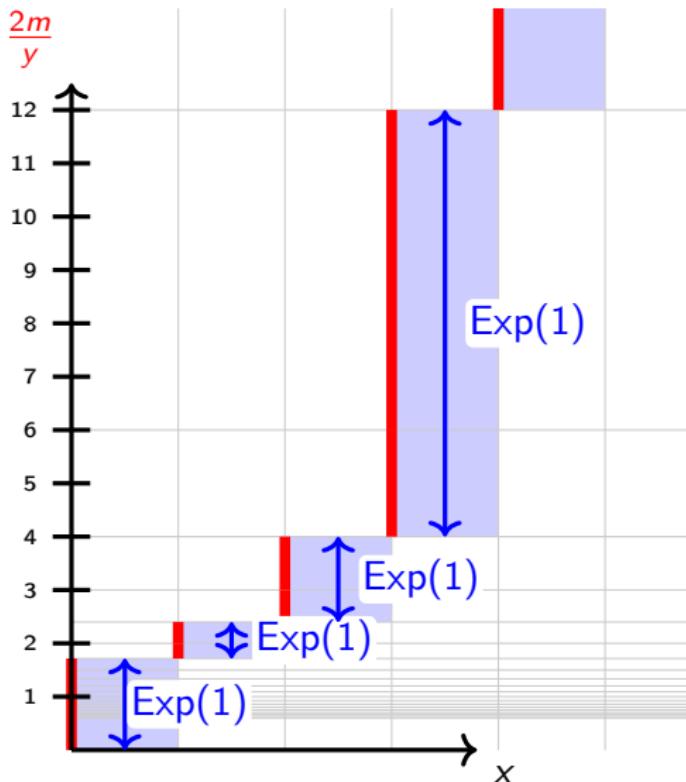
$$\Lambda^{(t)} \stackrel{\text{dist}}{\approx} \delta_x \times \text{Plancherel}(t)$$

then for $u > t$

$$\Lambda^{(u)} \stackrel{\text{dist}}{\approx} \text{Binom}\left(x, \sqrt{\frac{t}{u}}\right) \times \text{Plancherel}(u)$$

semi-projective plot, $m \rightarrow \infty$

$$\tau = \frac{2m}{y}$$



Corollary: the red line is a plot of the standard Poisson process

RSK
oooo

problems
ooo

theorem: bumping routes
ooooo

proof, the easy part
ooooo

proof, science fiction
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the end
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hydrodynamics of the insertion tableau $P(w)$

RSK
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problems
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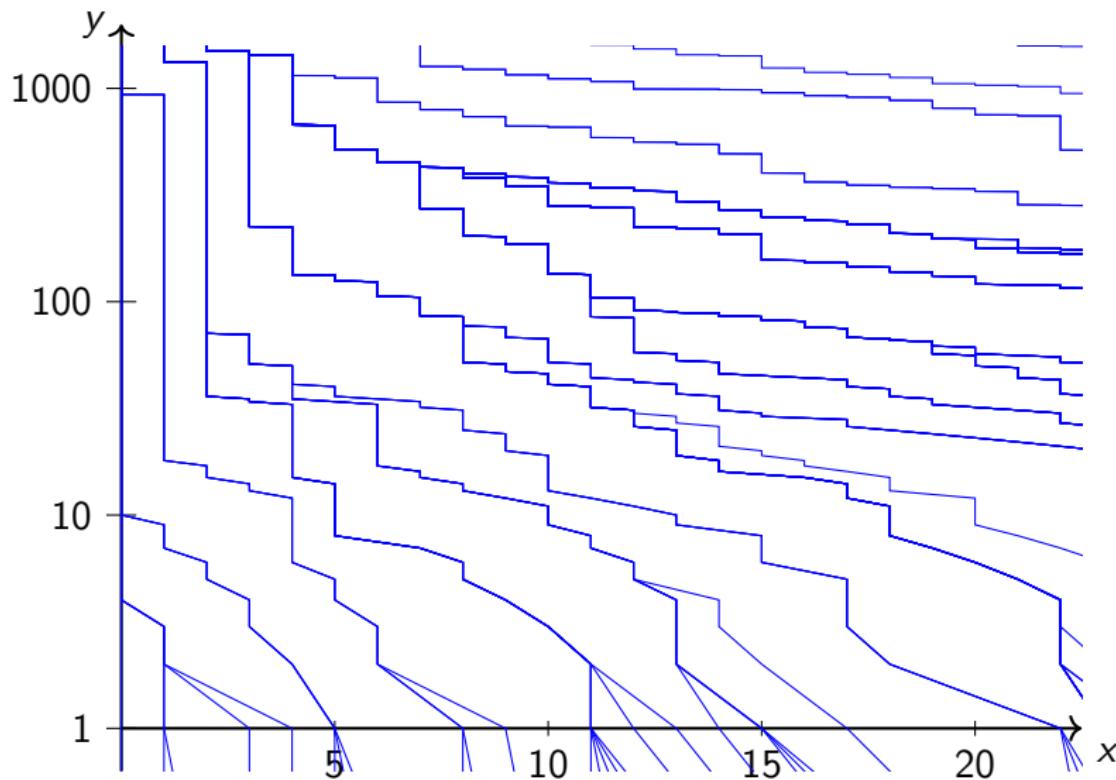
theorem: bumping routes
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proof, the easy part
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proof, science fiction
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the end
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open problems: bumping forest



RSK
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problems
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theorem: bumping routes
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proof, the easy part
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proof, science fiction
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the end
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Textbooks

The Surprising Mathematics of Longest Increasing Subsequences

Dan Romik



legal PDF file
available for free
on the author's
website



Mikołaj Marciniak,
Łukasz Maślanka,
Piotr Śniady
Poisson limit theorems
for the Robinson–Schensted
correspondence
and the Hammersley
multi-line process
arXiv:2005.13824



Mikołaj Marciniak,
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Piotr Śniady
Poisson limit
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arXiv:2005.14397



Dan Romik, Piotr Śniady.
Limit shapes of bumping routes in the Robinson–Schensted
correspondence.
Random Structures & Algorithms 48 (2016), no. 1, 171–182