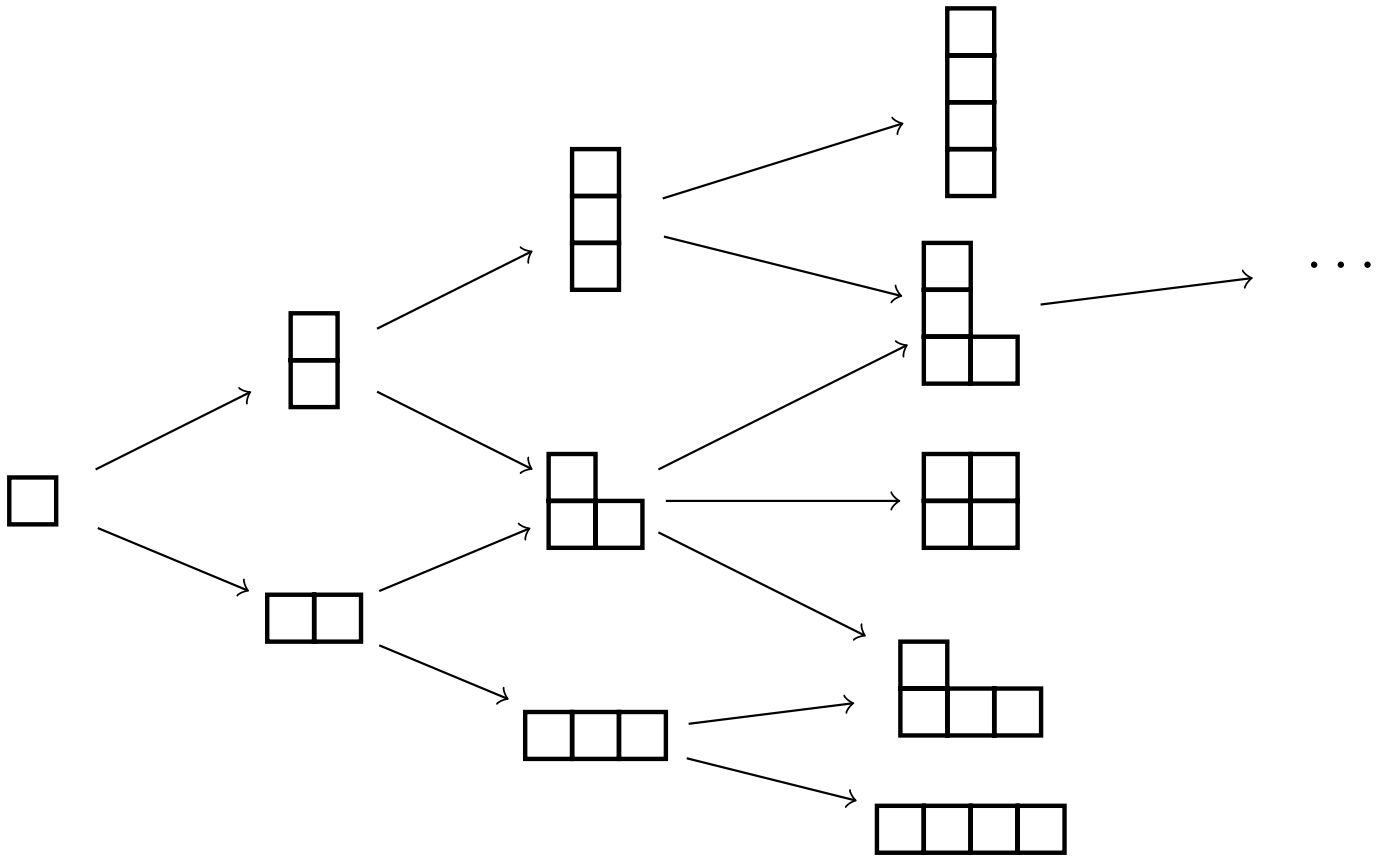
Trajectories of jeu-de-taquin and representations of the infinite symmetric group

joint work with Dan Romik

Piotr Śniady

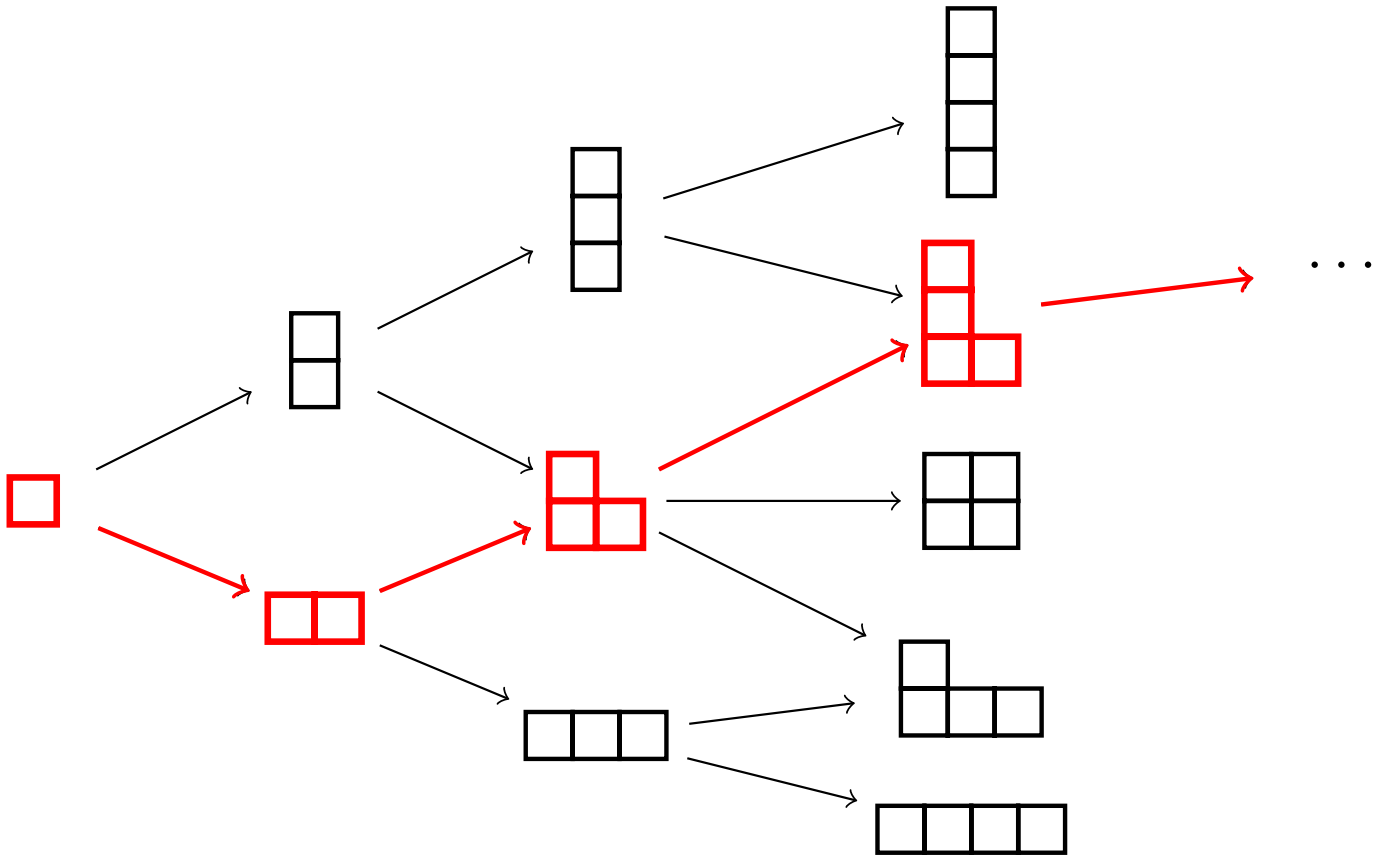
Polish Academy of Sciences
and
University of Wrocław

Young graph: irreducible representations of symmetric groups $S_1 \subset S_2 \subset S_3 \subset \dots$



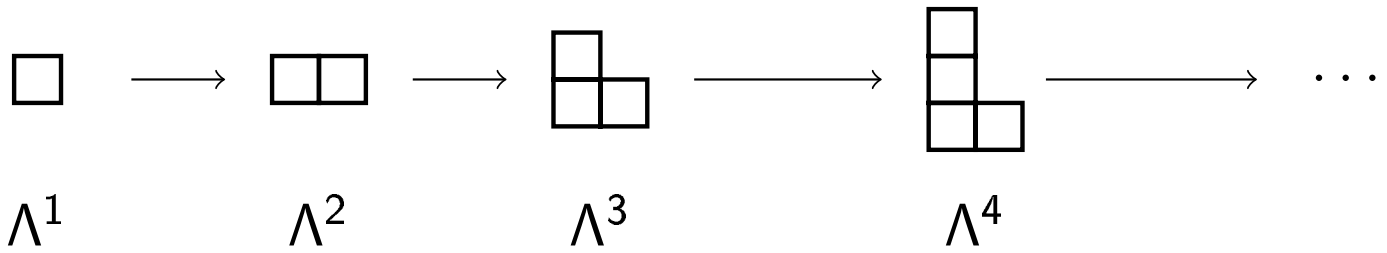
Tool for studying S_∞

Young graph: irreducible representations of symmetric groups $S_1 \subset S_2 \subset S_3 \subset \dots$

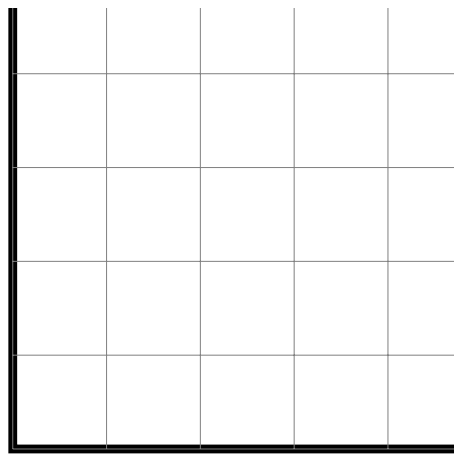


Tool for studying S_∞

paths in Young graph $\leftrightarrow$ tableaux



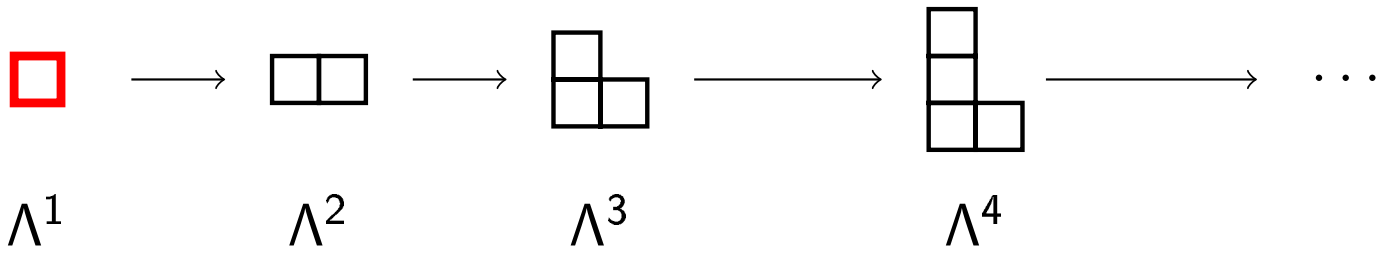
infinite path in Young graph



infinite tableau

Ω - set of infinite tableaux / set of infinite paths

paths in Young graph $\leftrightarrow$ tableaux



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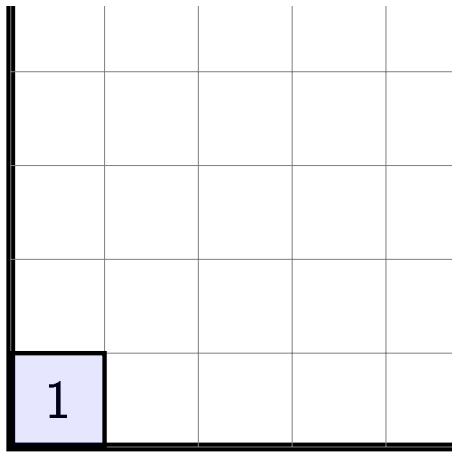

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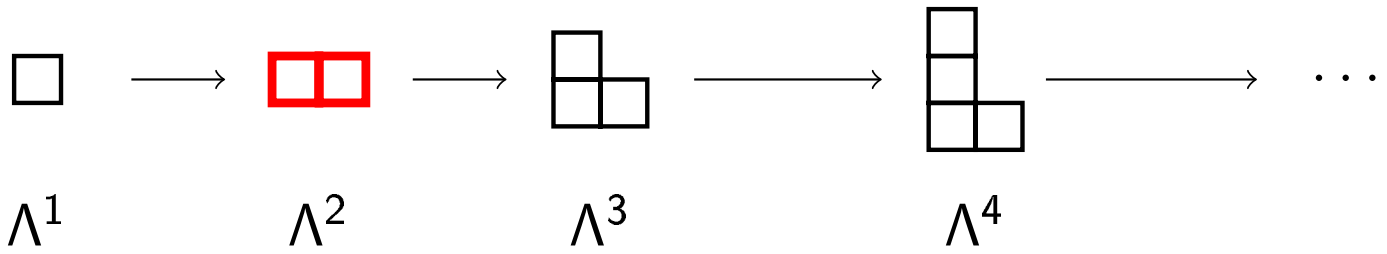

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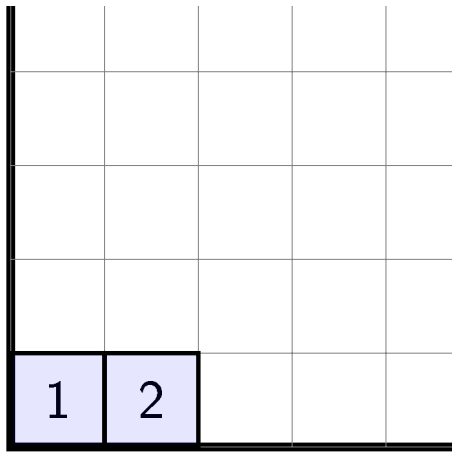

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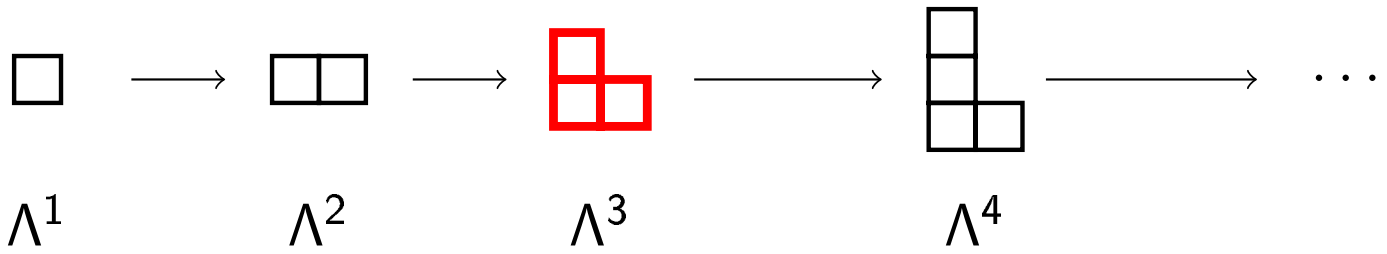

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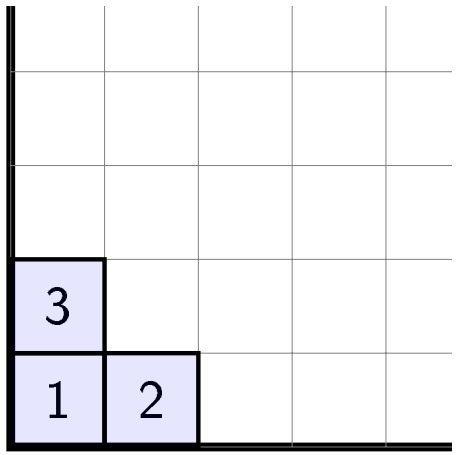

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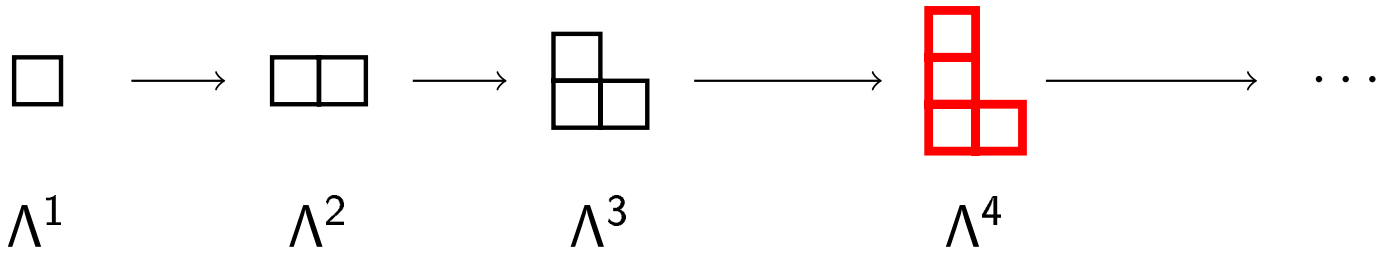

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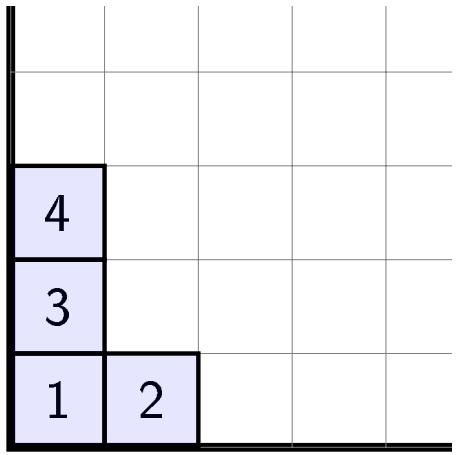

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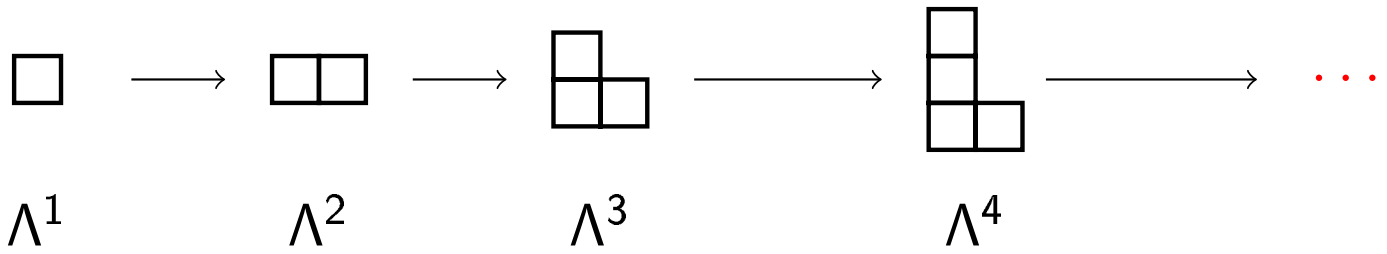


infinite path in Young graph

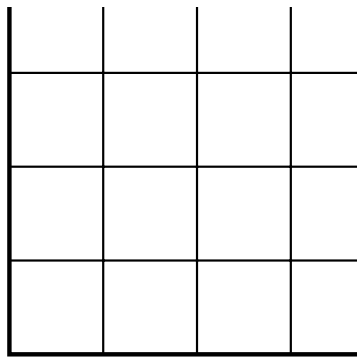
⋮		⋮		
6	15	21	24	
4	12	17	19	⋯
3	5	8	11	
1	2	7	9	⋯

infinite tableau

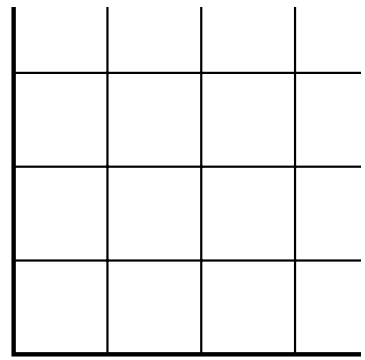
Ω - set of infinite tableaux / set of infinite paths

infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\overset{\text{RSK}}{\mapsto}$ recording tableau



insertion tableau



recording tableau

F O X D R P B Z U L G E A T W N S M Y V C J H Q I K

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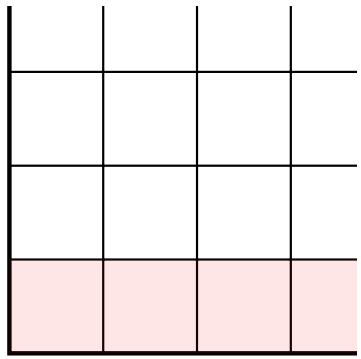

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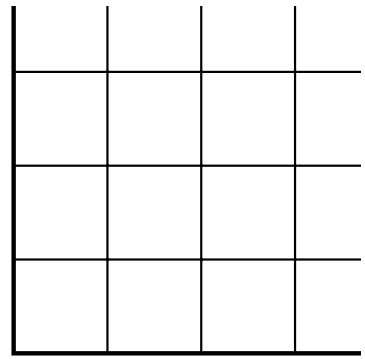

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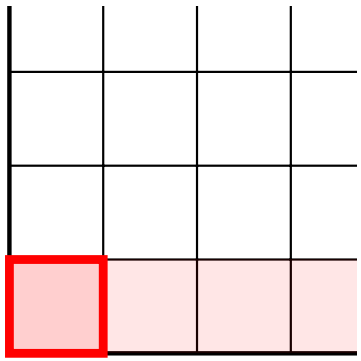

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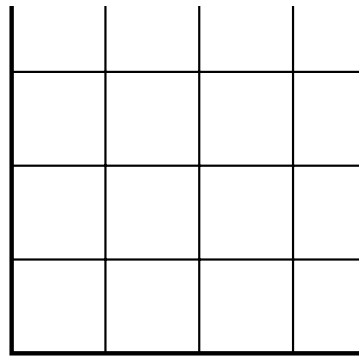

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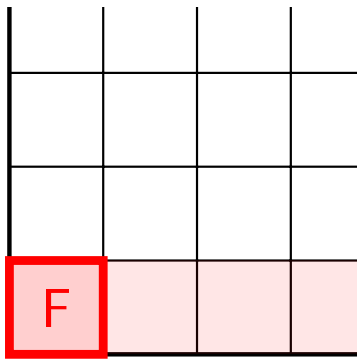

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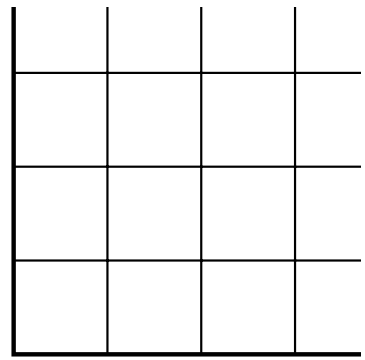

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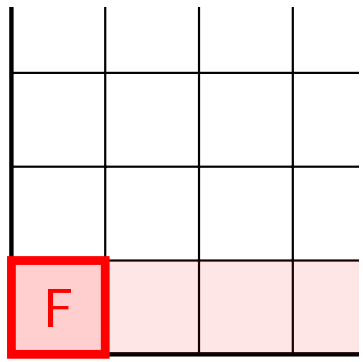

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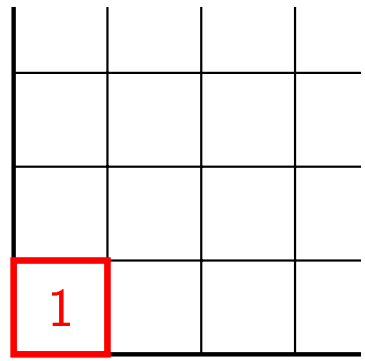

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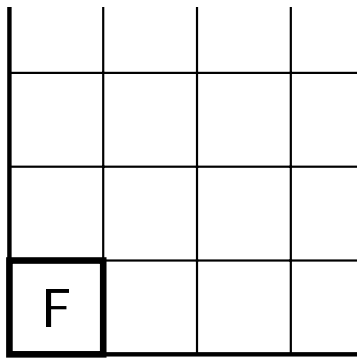

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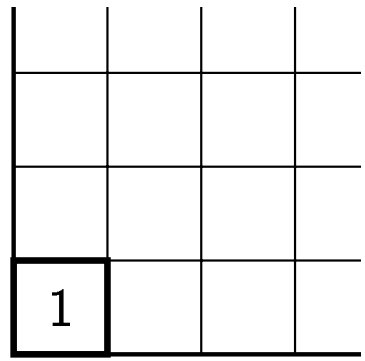

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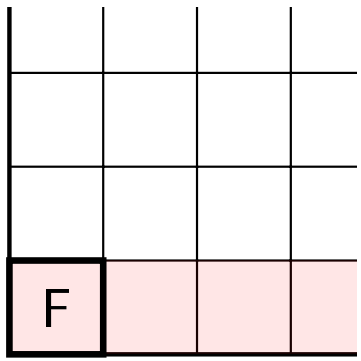

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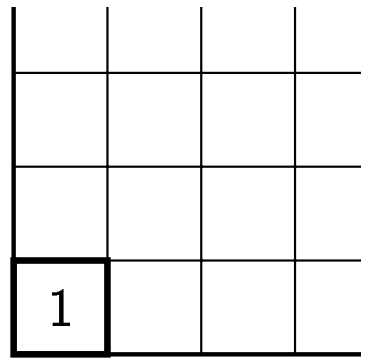

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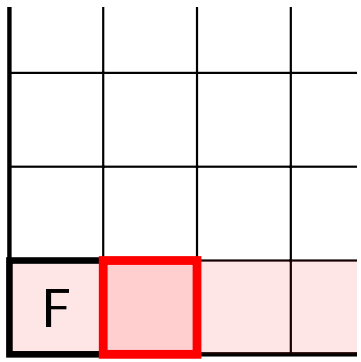

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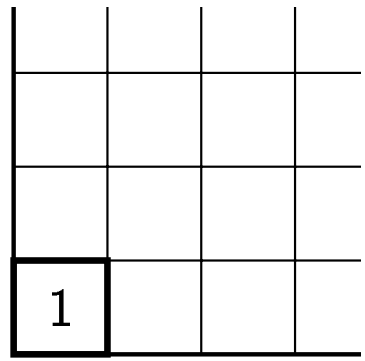

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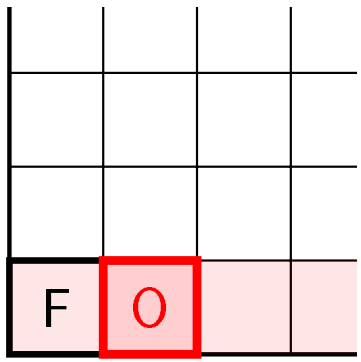

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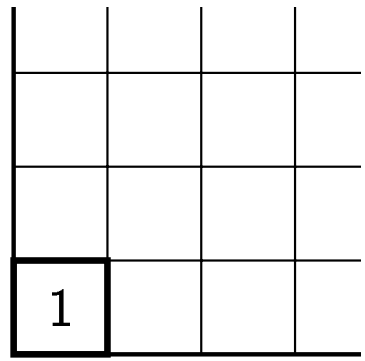

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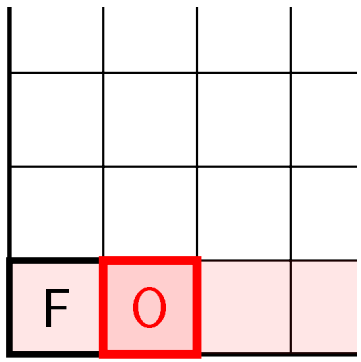

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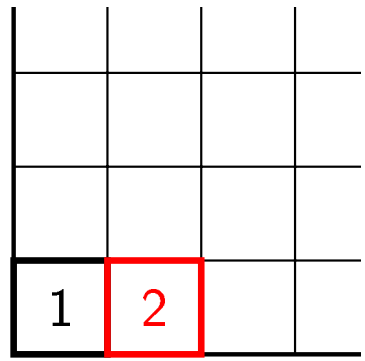

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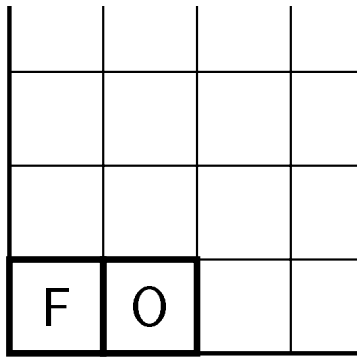

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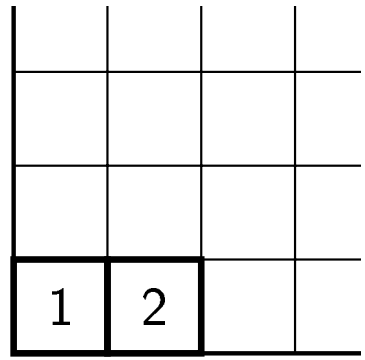

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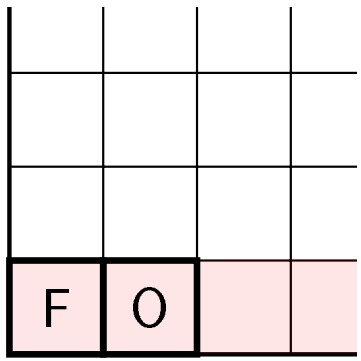

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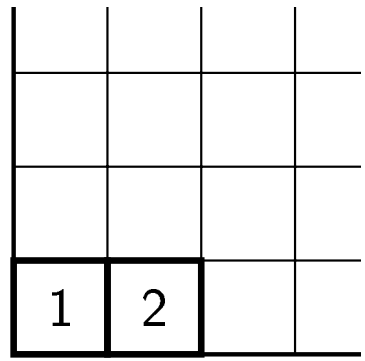

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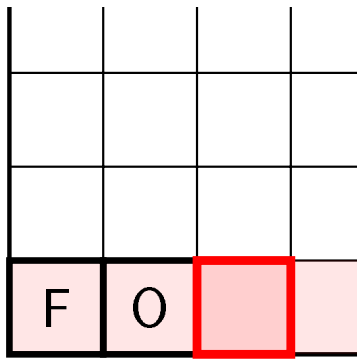

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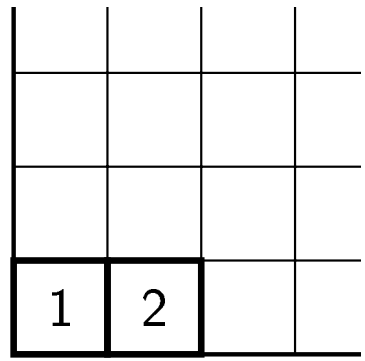

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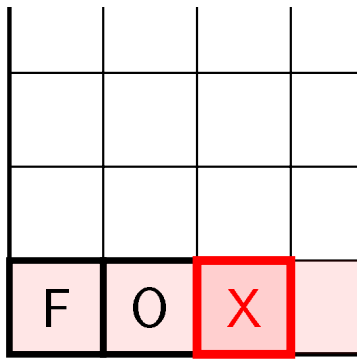

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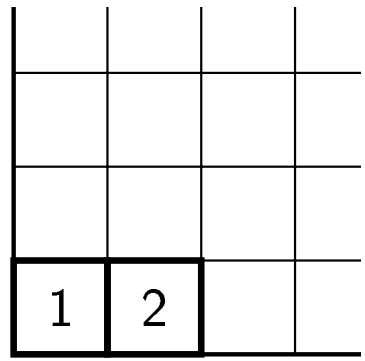

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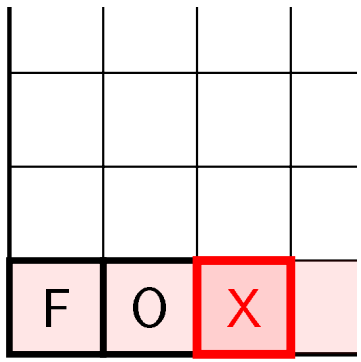

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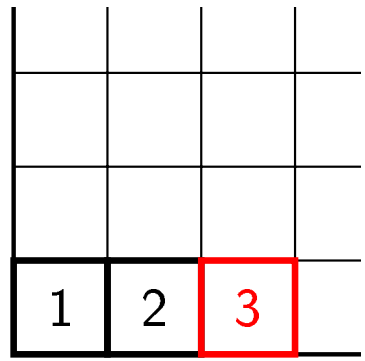

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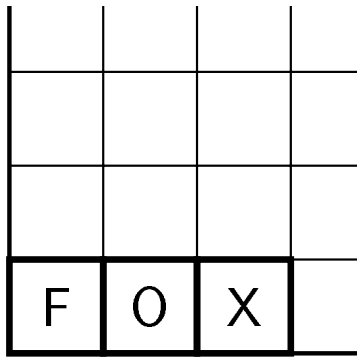

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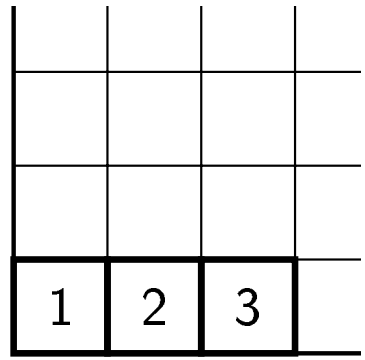

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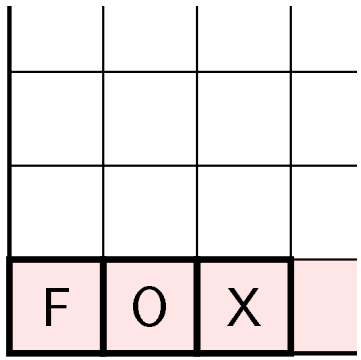
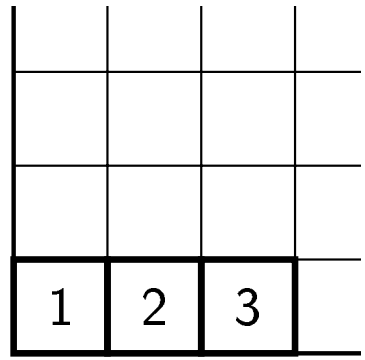
infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\overset{\text{RSK}}{\mapsto}$ recording tableau



insertion tableau



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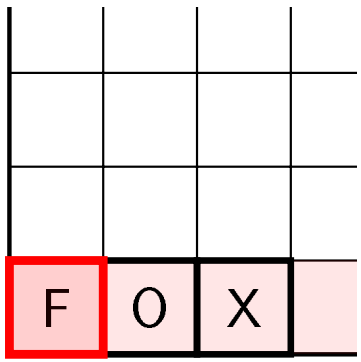

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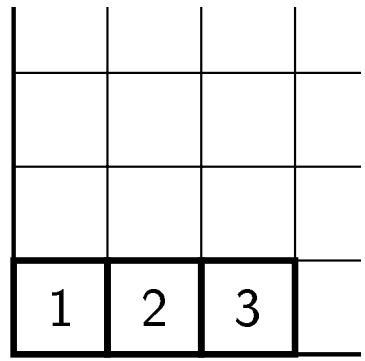

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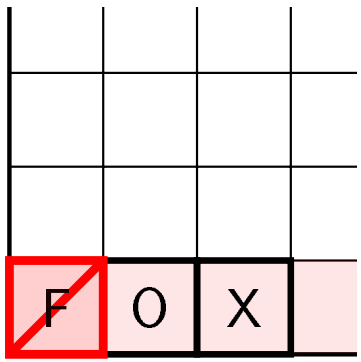

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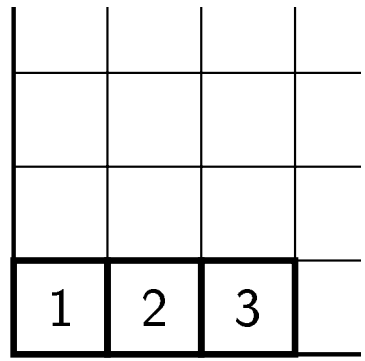

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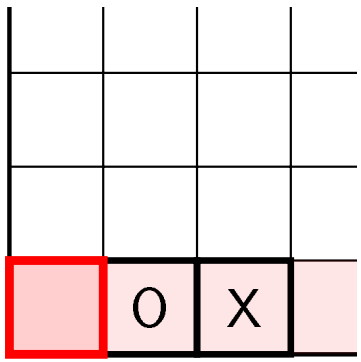

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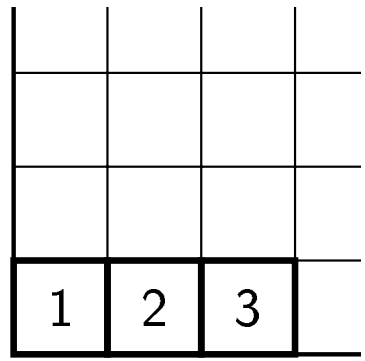

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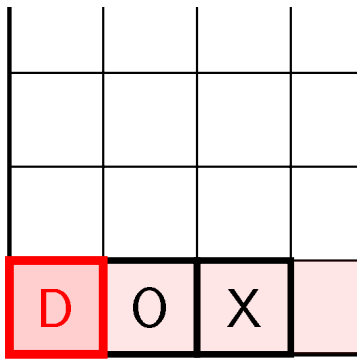

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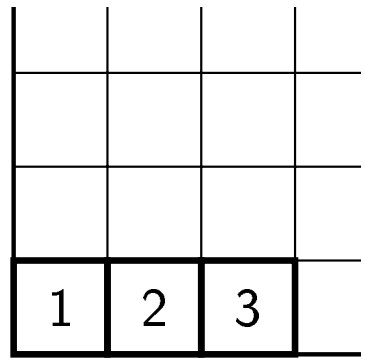

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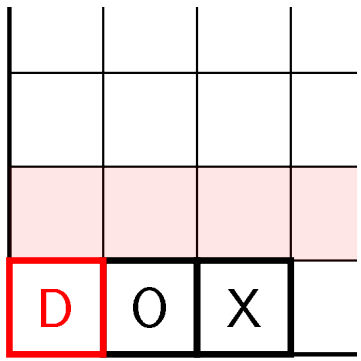

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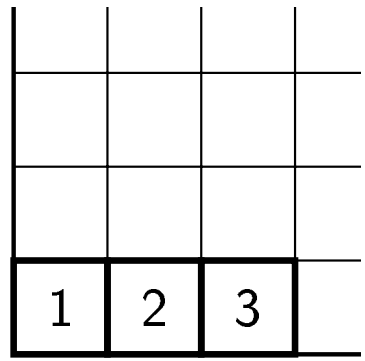

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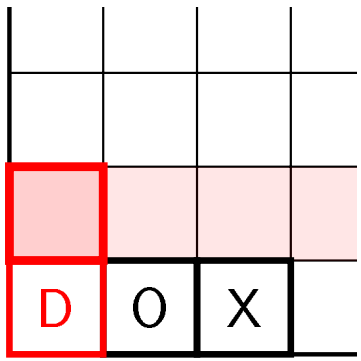

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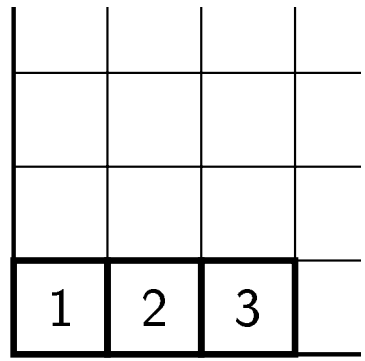

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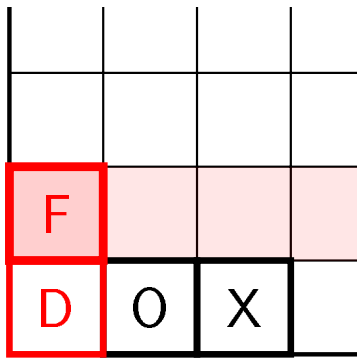

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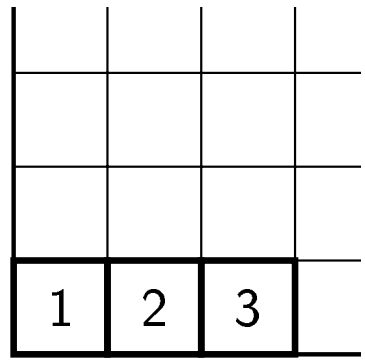

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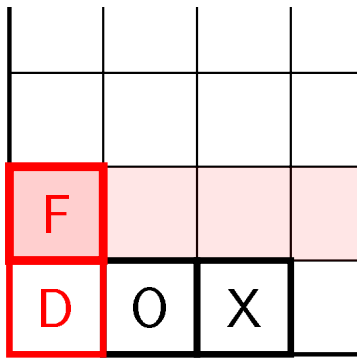

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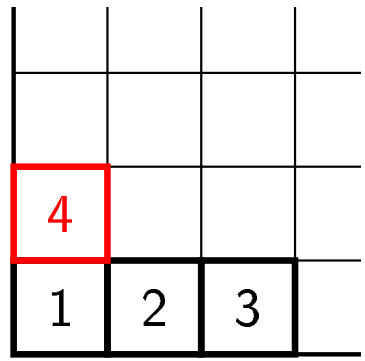

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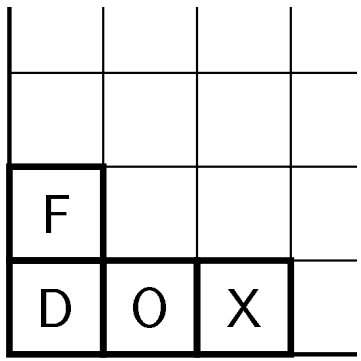

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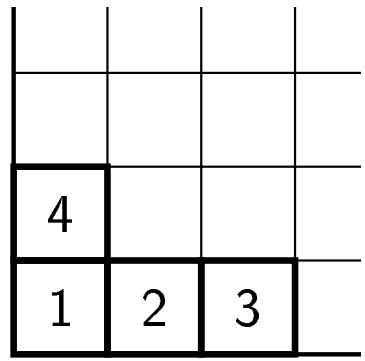

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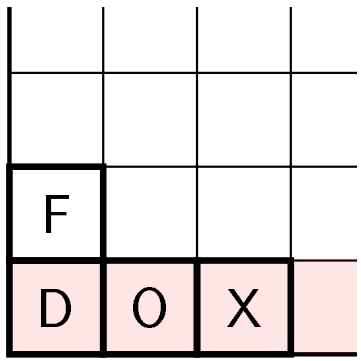

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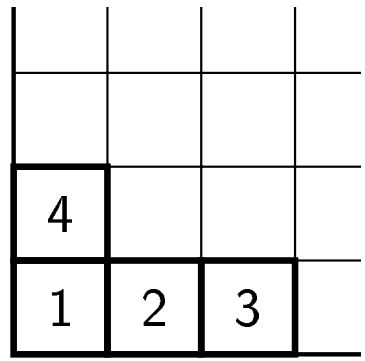

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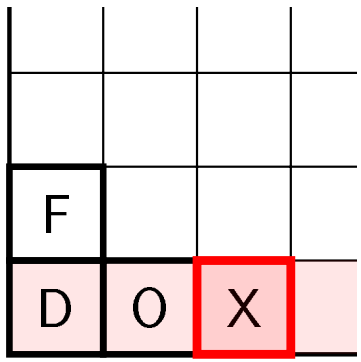

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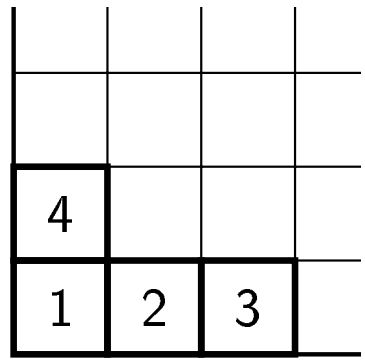

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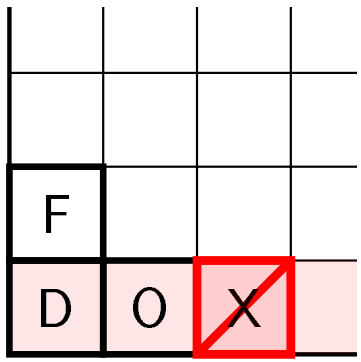

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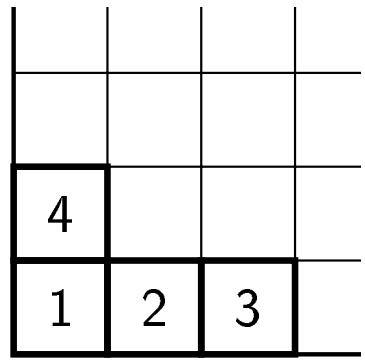

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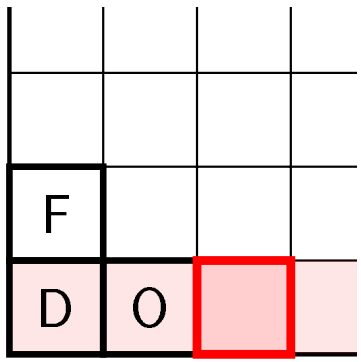

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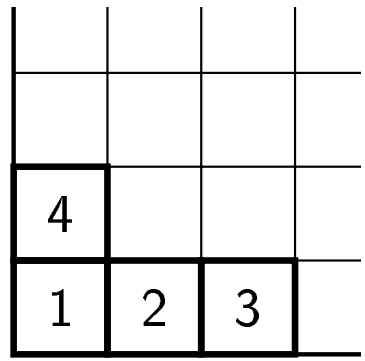

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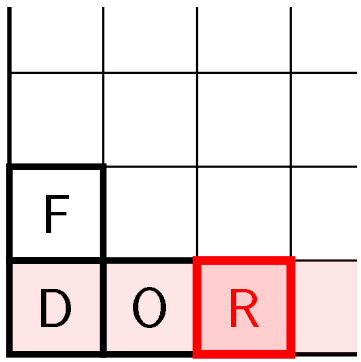

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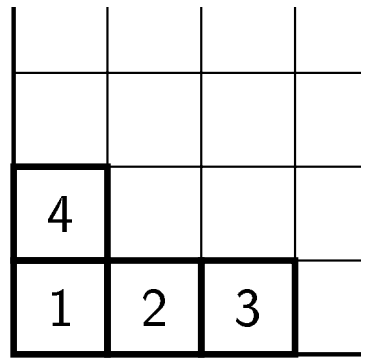

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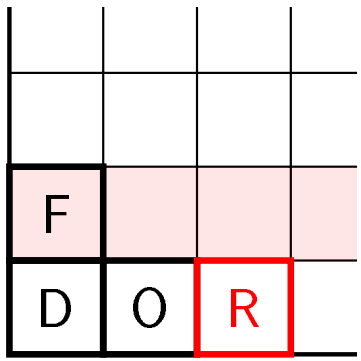

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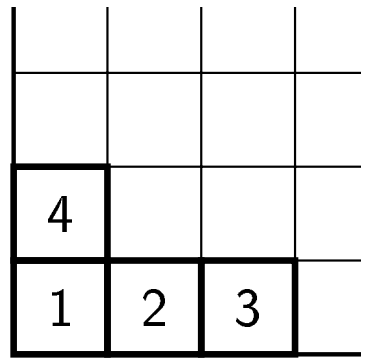

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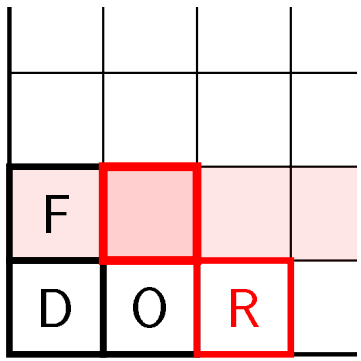

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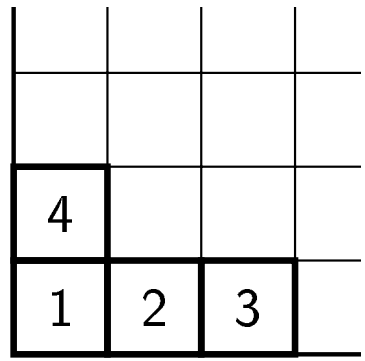

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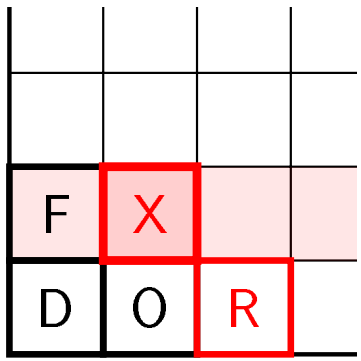

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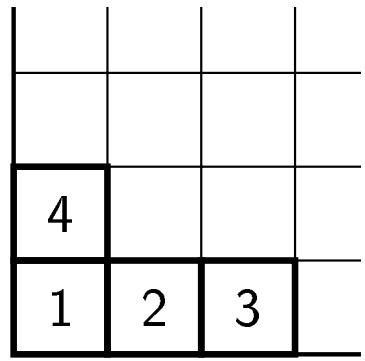

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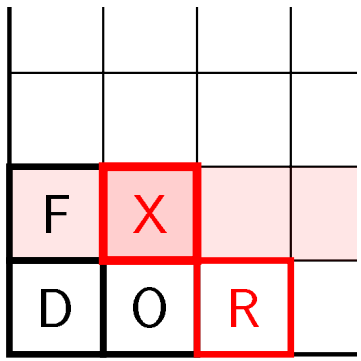

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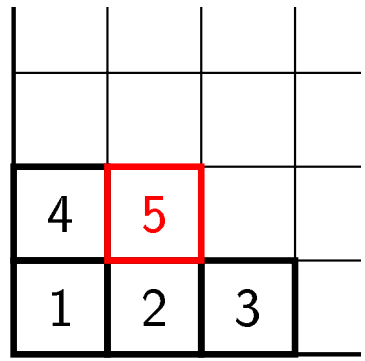

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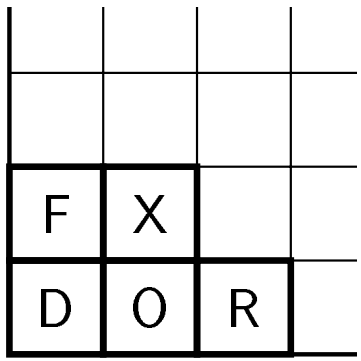

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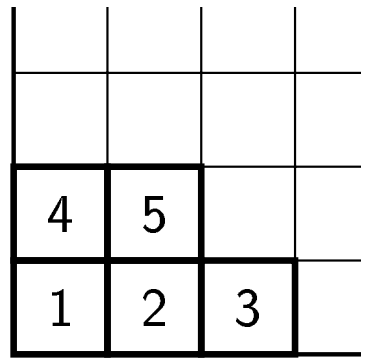

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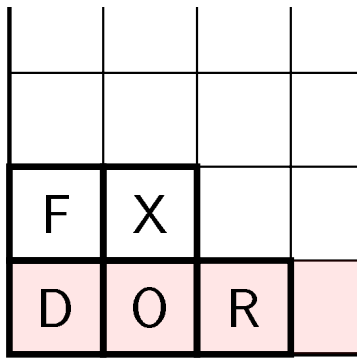
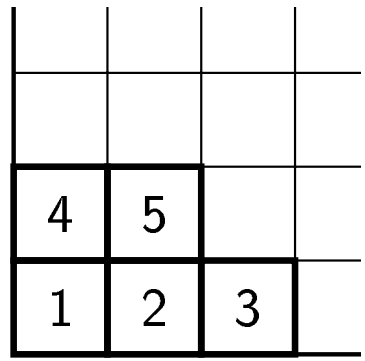


insertion tableau



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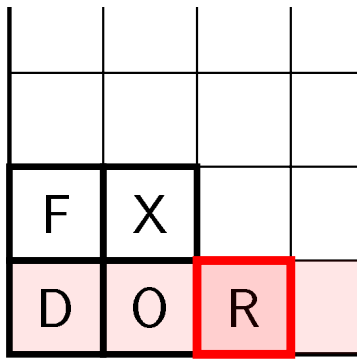

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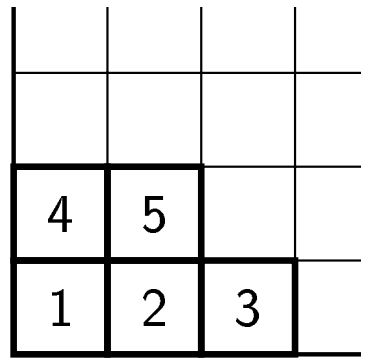

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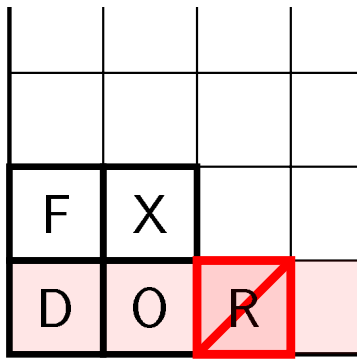

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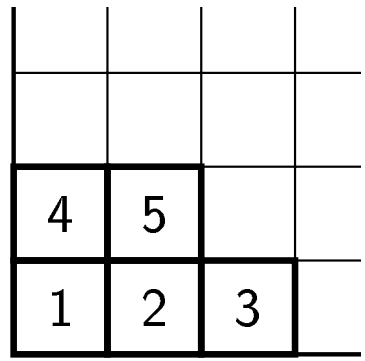

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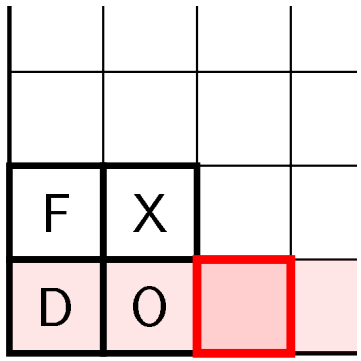

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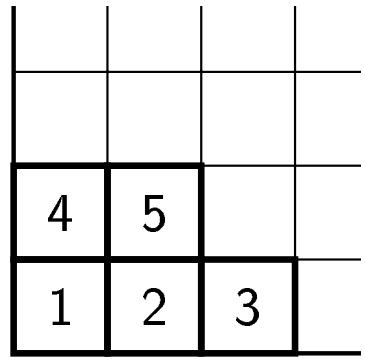

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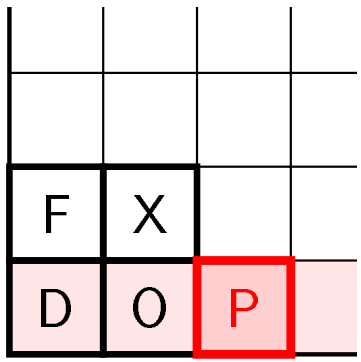

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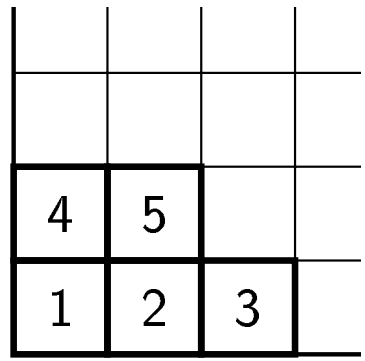

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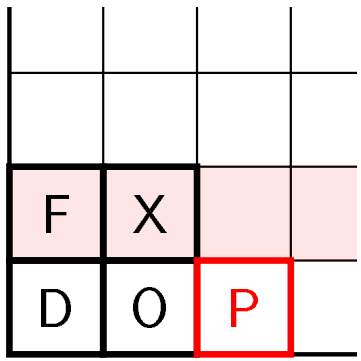

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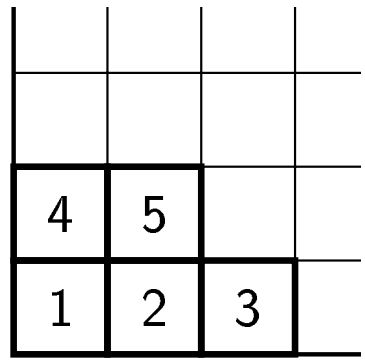

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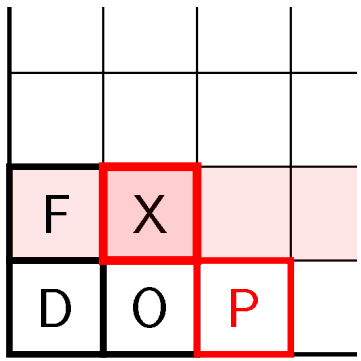

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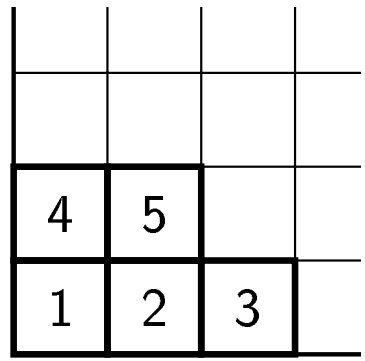

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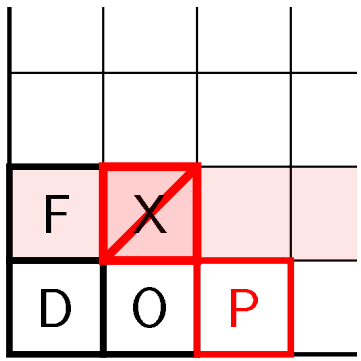

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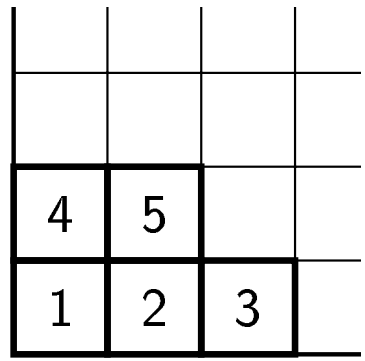

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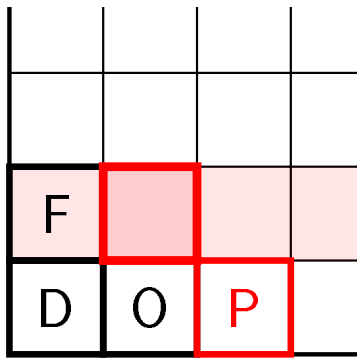

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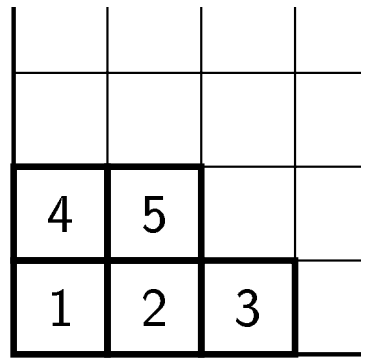

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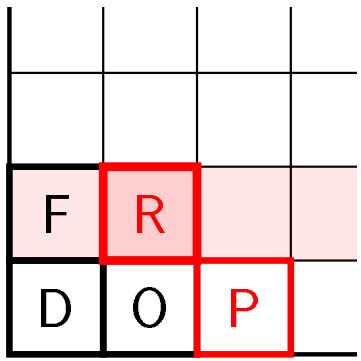

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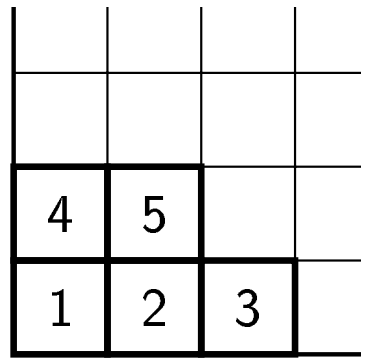

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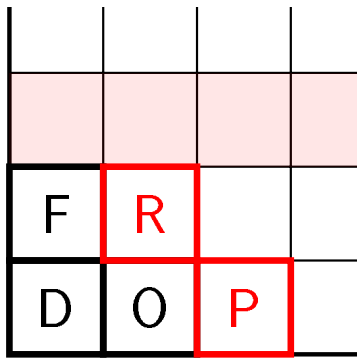

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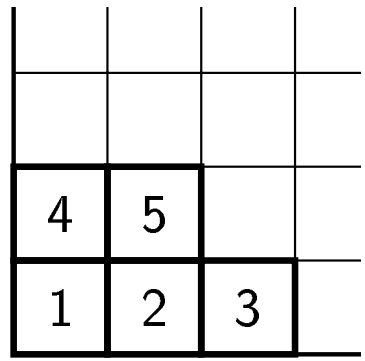

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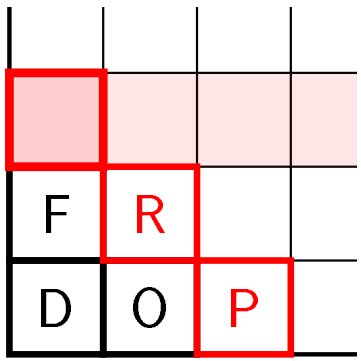

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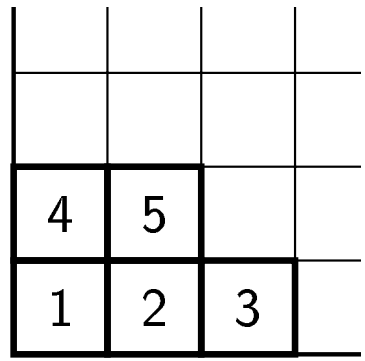

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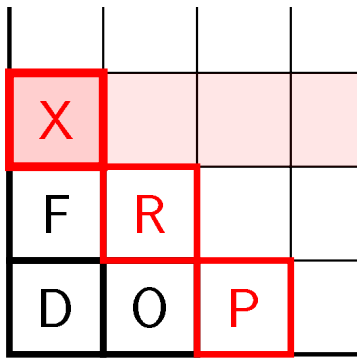

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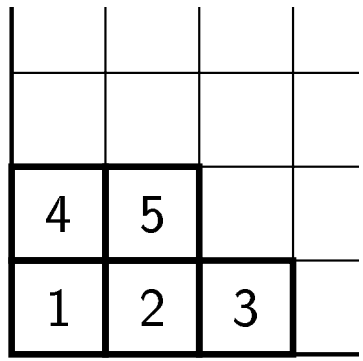

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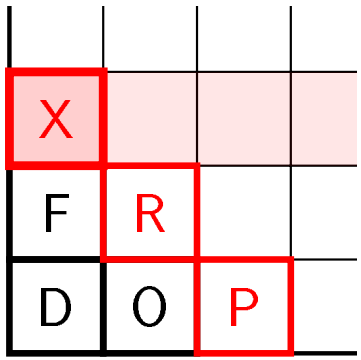

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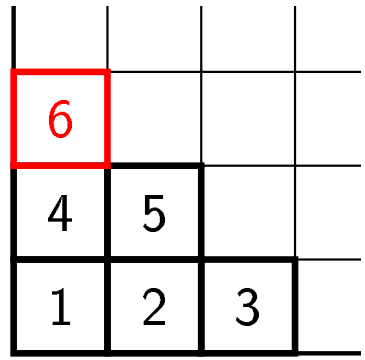

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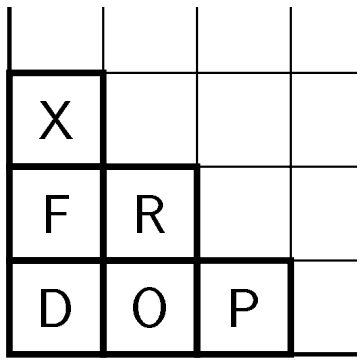

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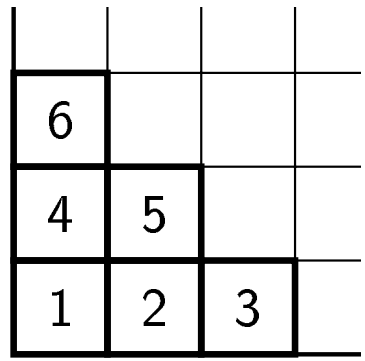

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F	L	N	T
D	G	M	S
B	E	J	Q
A	C	H	I

insertion tableau

7	16	22	25
6	10	14	24
4	5	9	17
1	2	3	8

recording tableau

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insertion tableau recording tableau

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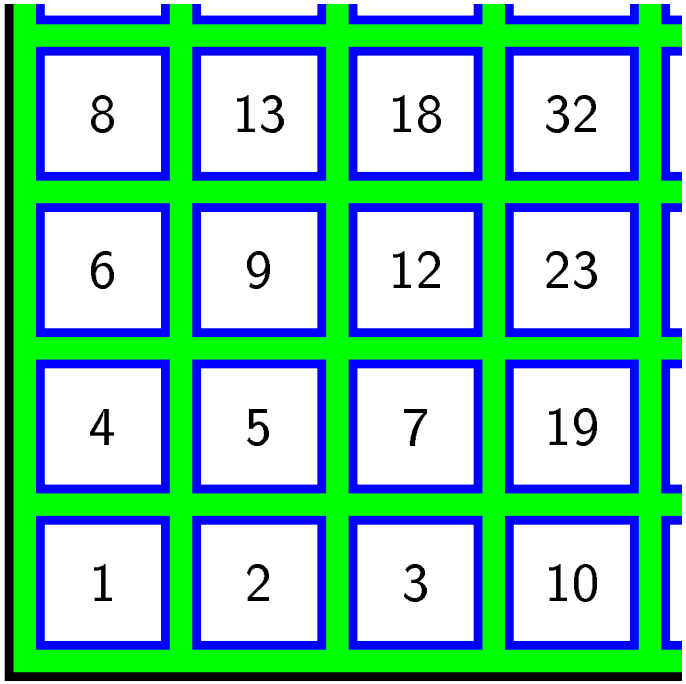
insertion tableau

recording tableau

F O X D R P B Z U L G E A T W N S M Y V C J H Q I K

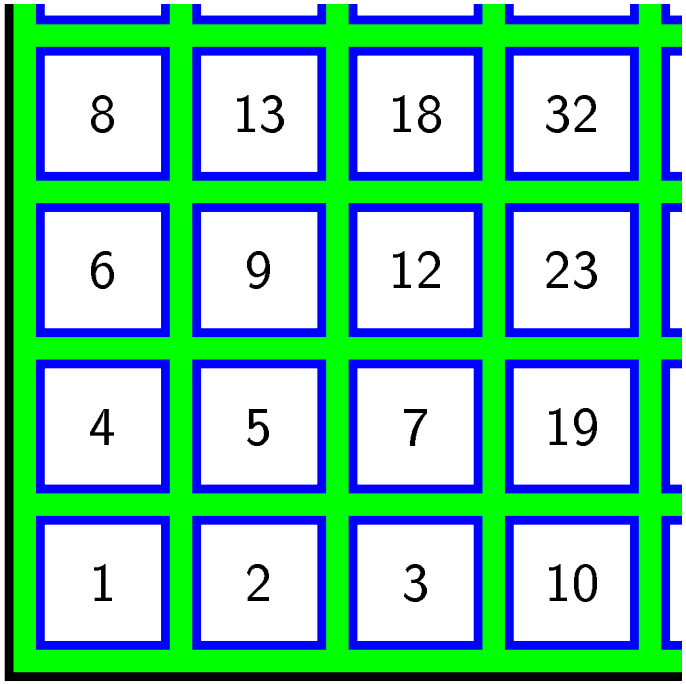
if $X_0, X_1, \ldots$ are i.i.d. $U(0,1)$ random variables then

$$\text{RSK}(X_0, X_1, \ldots) \overset{\text{distribution}}{=} \text{Plancherel measure}$$



Jeu de taquin

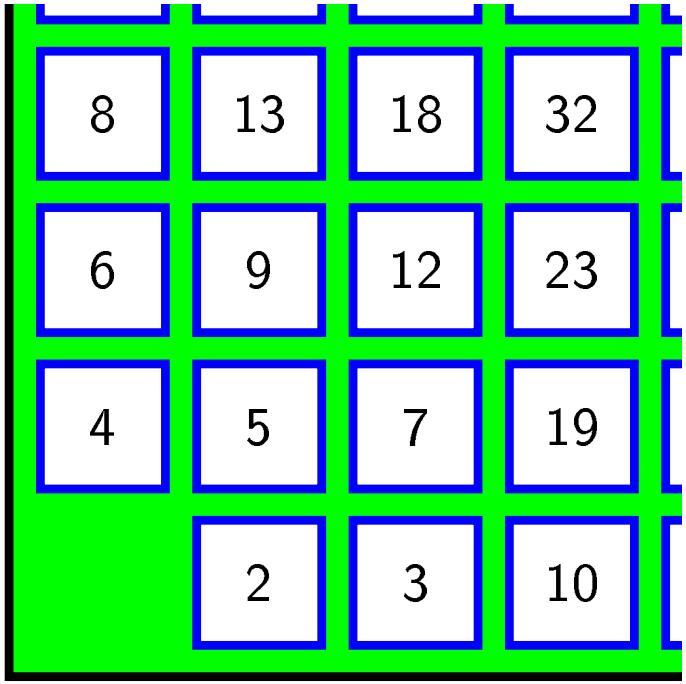
① start with $t \in \Omega$,



Jeu de taquin

① start with $t \in \Omega$,

② remove corner box,



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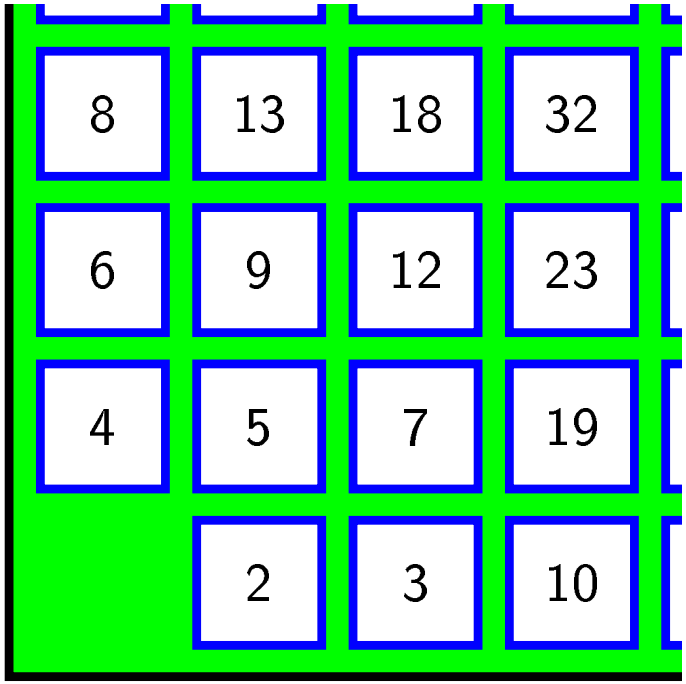

Jeu de taquin

① start with $t \in \Omega$,
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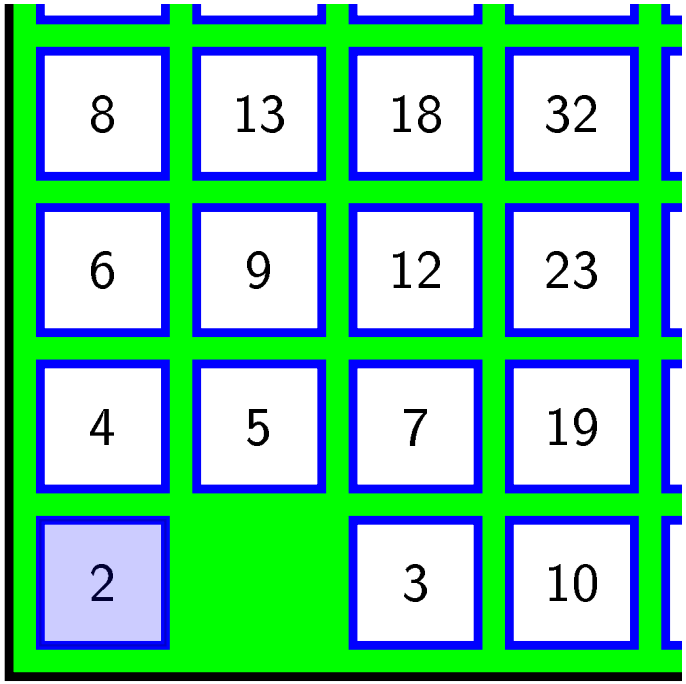
③ sliding,



Jeu de taquin

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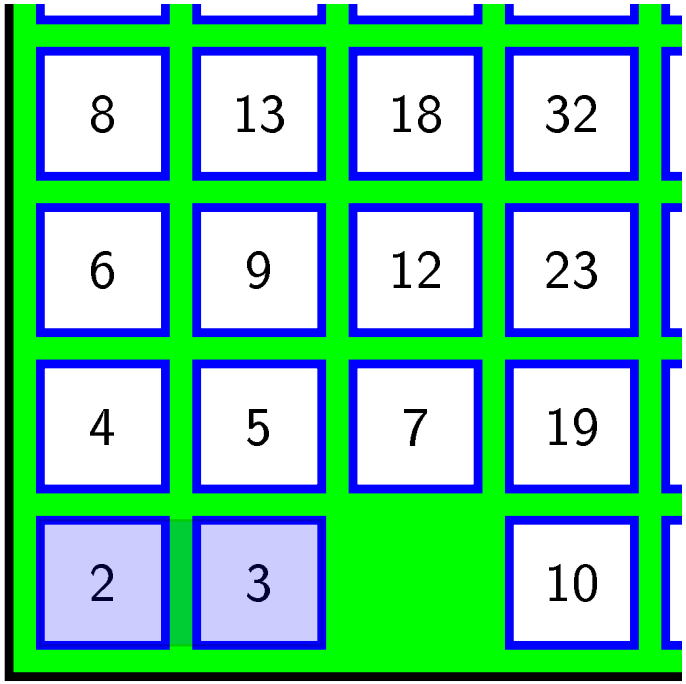

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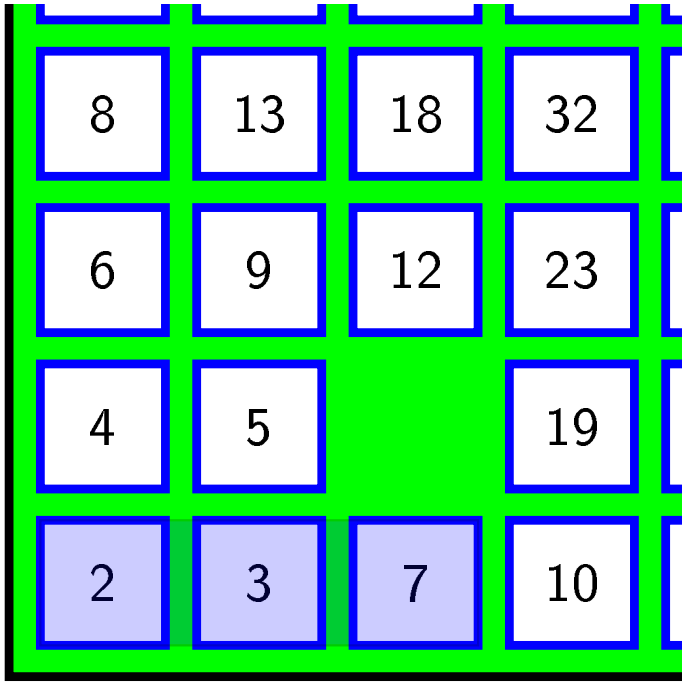

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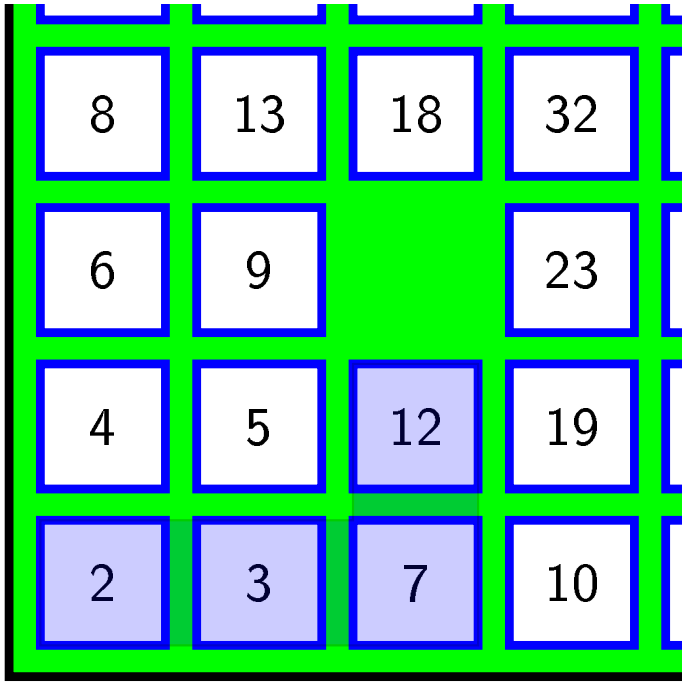

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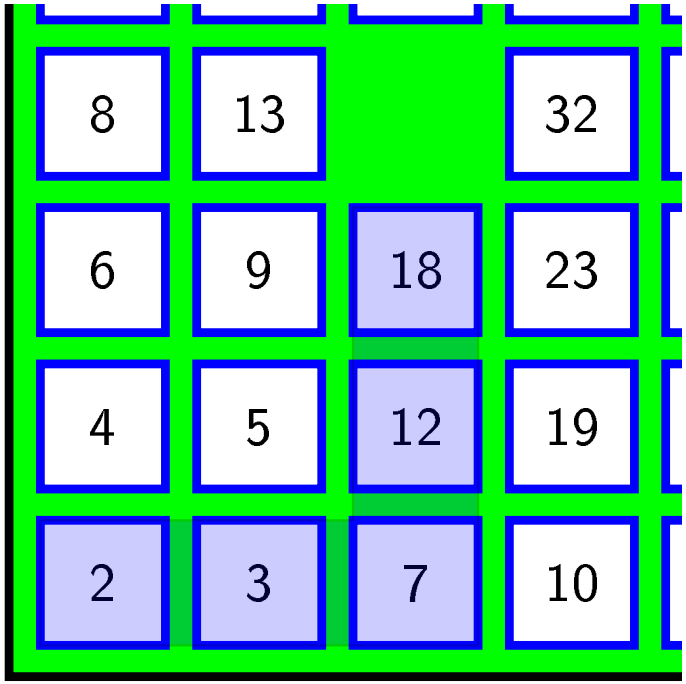

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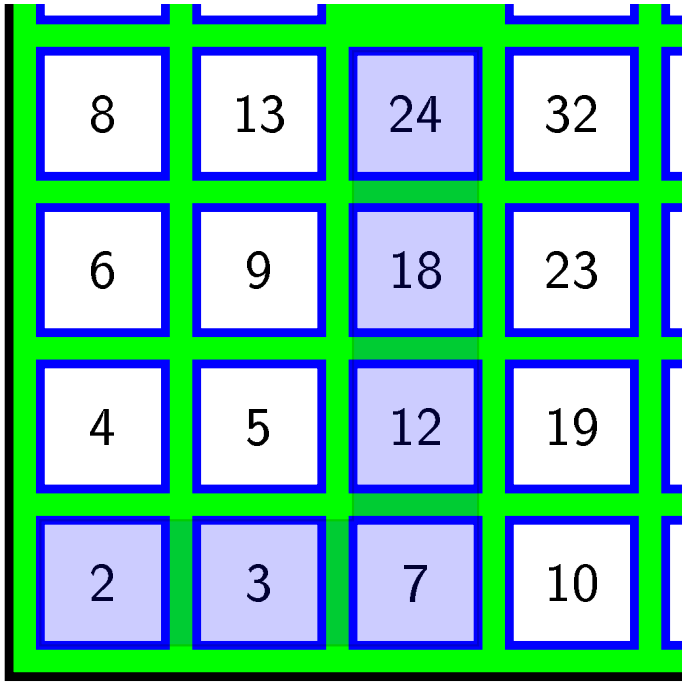

Jeu de taquin

① start with $t \in \Omega$,
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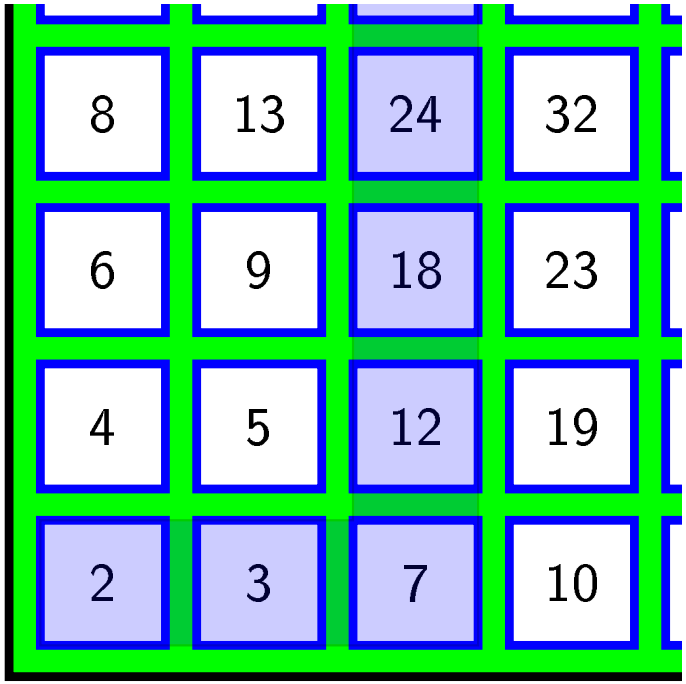
③ sliding,



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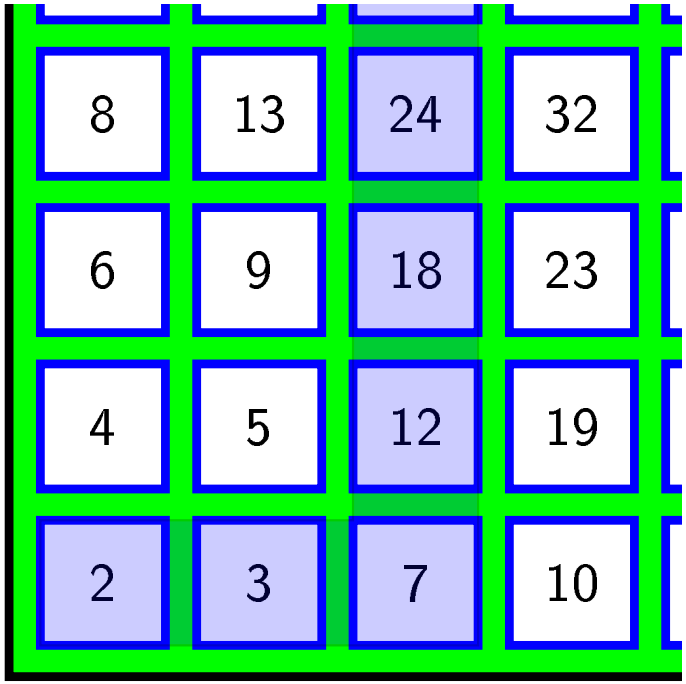

Jeu de taquin

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② remove corner box,
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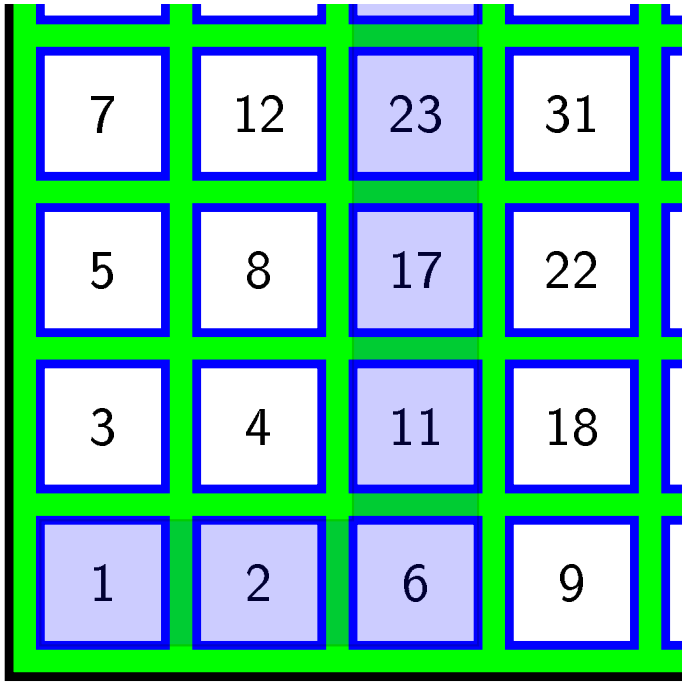
④ subtract 1 from all boxes



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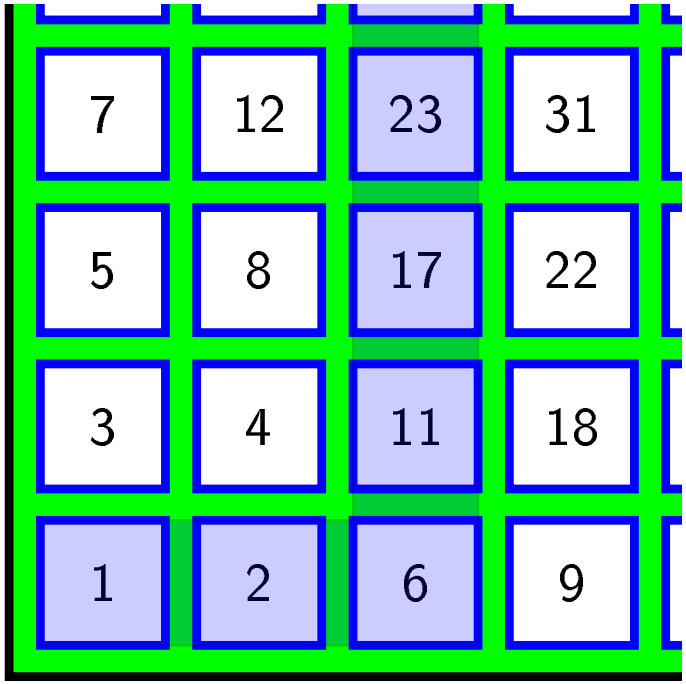

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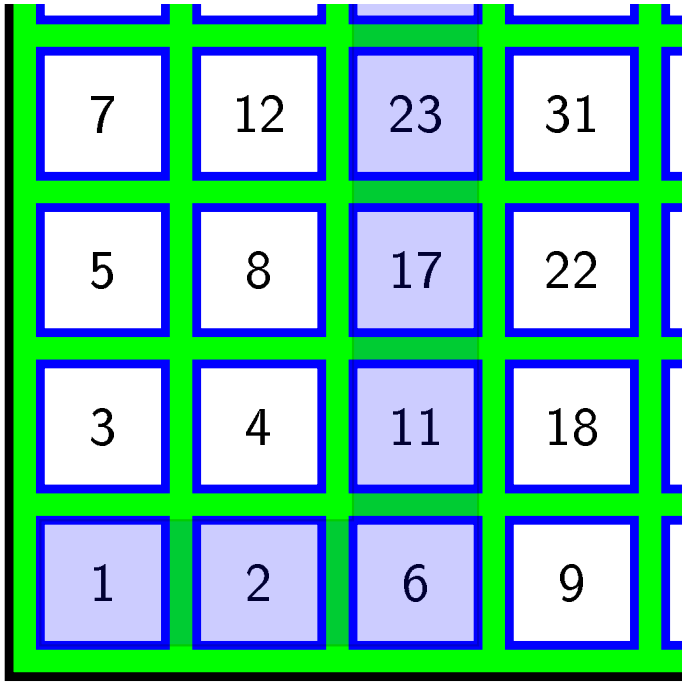
output:

- new tableau $J(t)$,
- trajectory $\mathbf{c}(t) = (c_1, c_2, \dots)$



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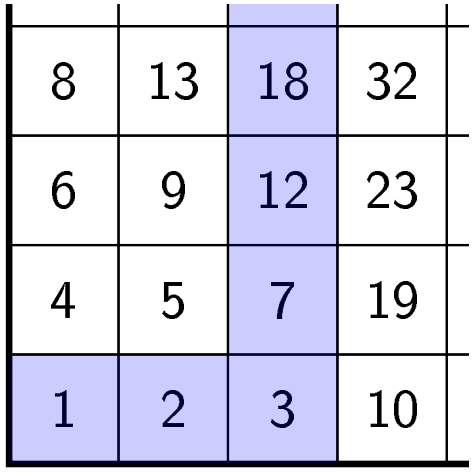
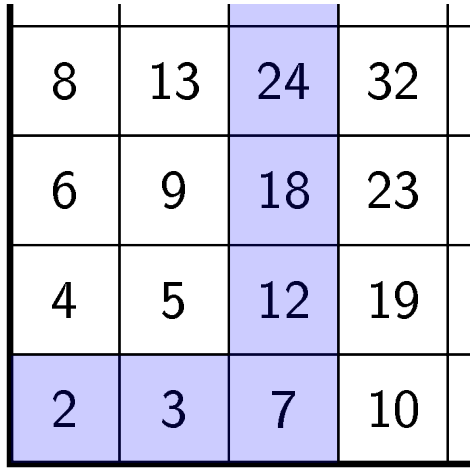
'how representation of $S_{\{1,2,\dots\}}$ is related to its restriction to $S_{\{2,3,\dots\}}$?'

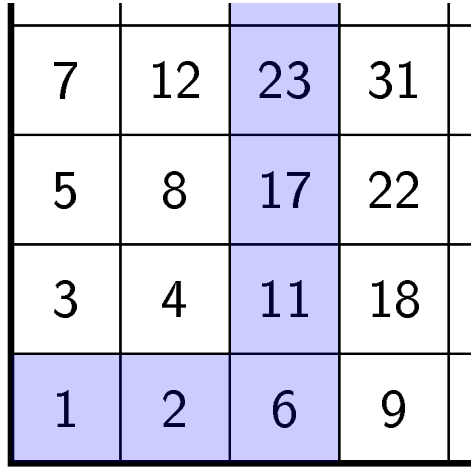
Jeu de taquin - overview



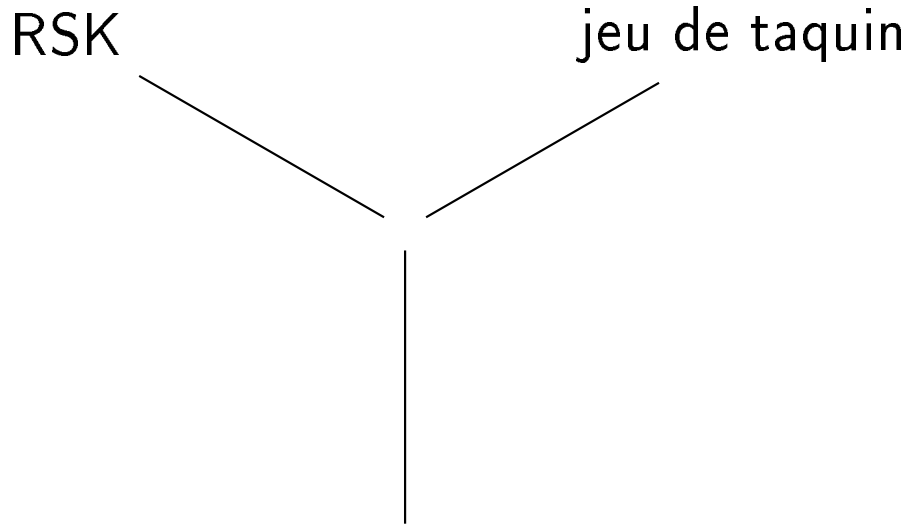
original tableau t



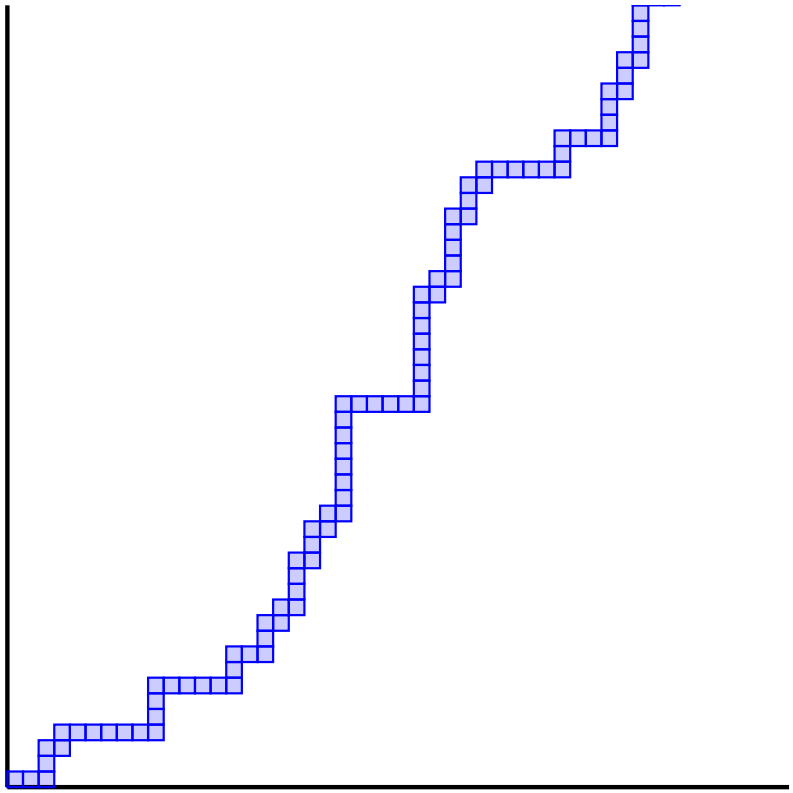
outcome of slidings



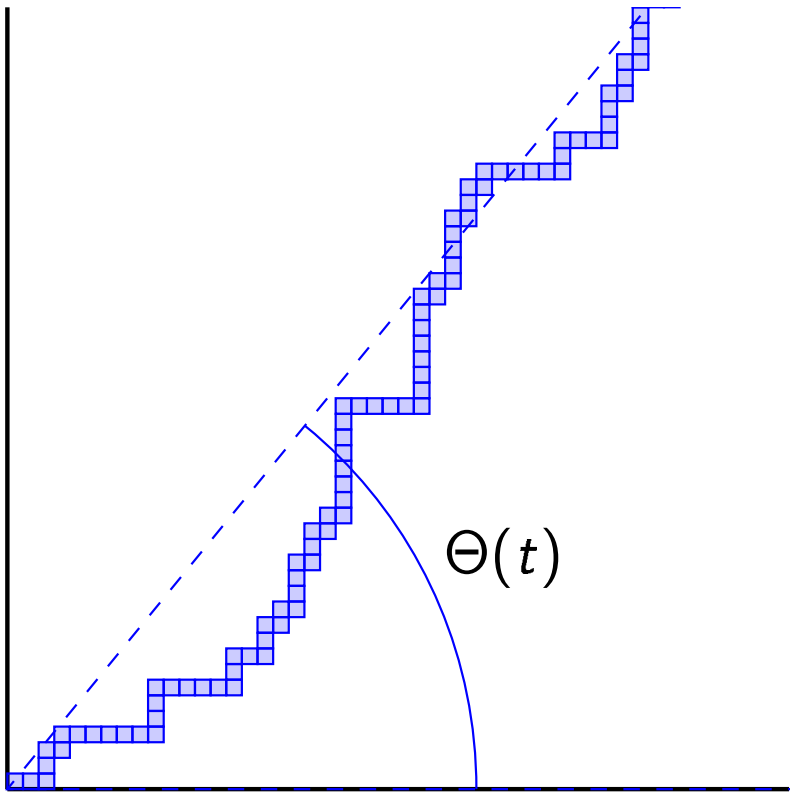
new tableau $J(t)$


RSK
jeu de taquin
Littlewood-Richardson rule
$V^\lambda \otimes V^\mu \uparrow_{S_n \times S_k}^{S_{n+k}} = \bigoplus_\nu c_{\lambda\mu}^\nu V^\nu$

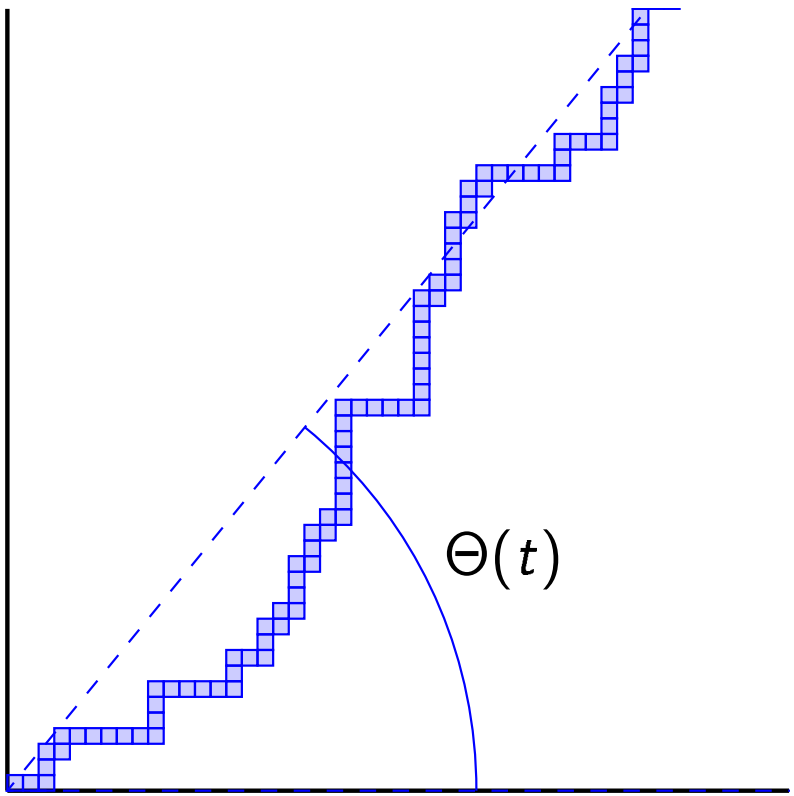
trajectories of jeu de taquin



trajectories of jeu de taquin



trajectories of jeu de taquin



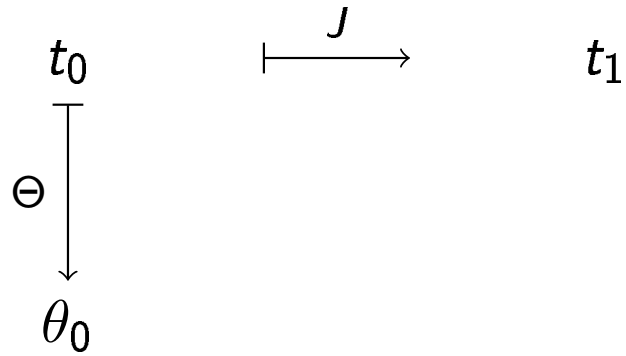
if $t \in \Omega$ is random,
Plancherel distributed

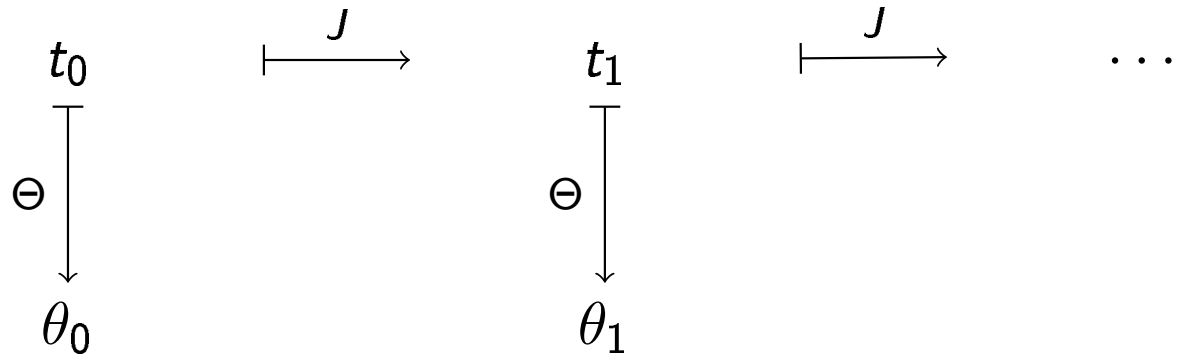
then its jdt trajectory $\mathbf{c}(t)$
is almost surely asymptotically
a straight line,

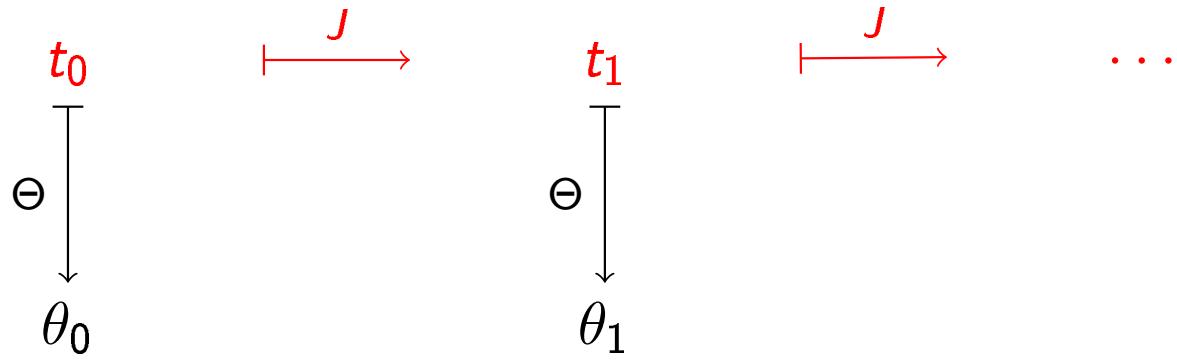
i.e.

$$\lim_{k\to\infty} \frac{c_k}{\|c_k\|} = \big(\cos\Theta(t), \sin\Theta(t)\big)$$

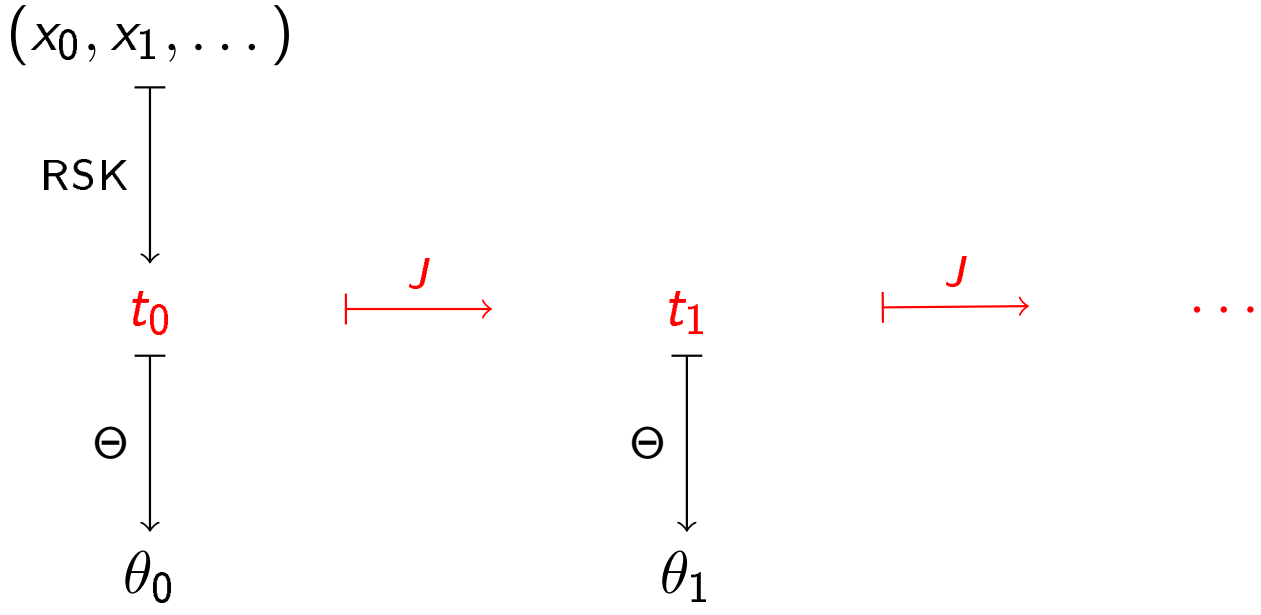
exists almost surely


t_0
J
t_1
Θ
θ_0


t_0
J
t_1
J
$\cdots$
$\ominus$
$\ominus$
θ_0
θ_1



jeu de taquin dynamical system $\left(\Omega, \text{Plancherel}, J\right)$



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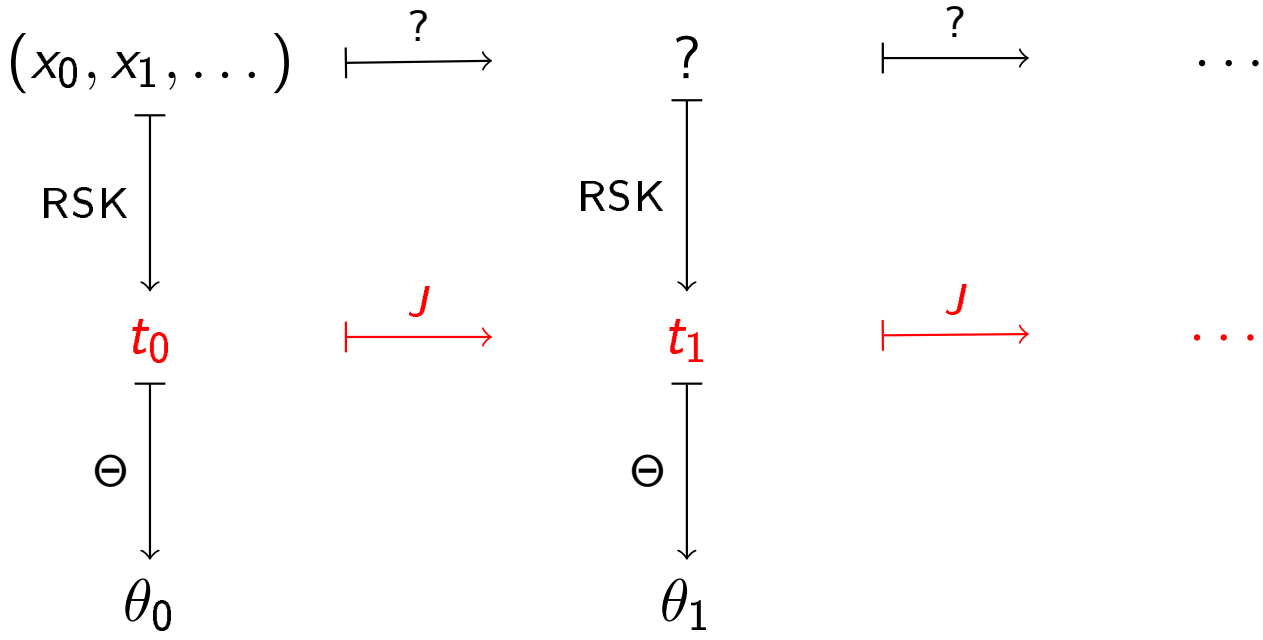

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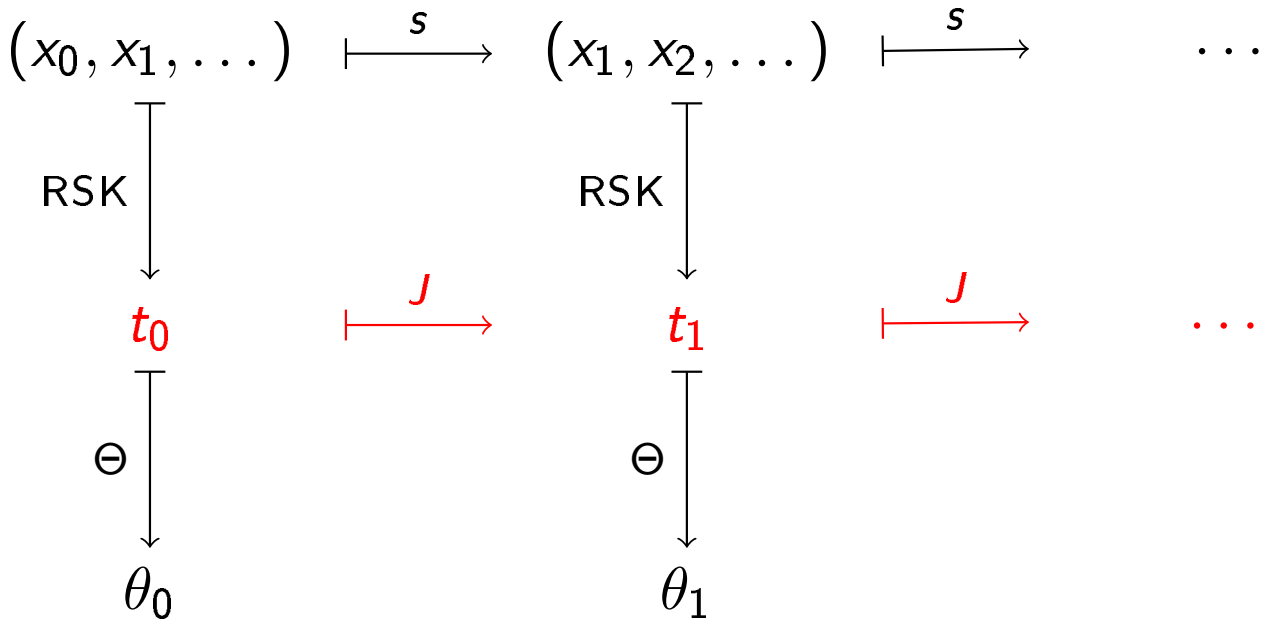
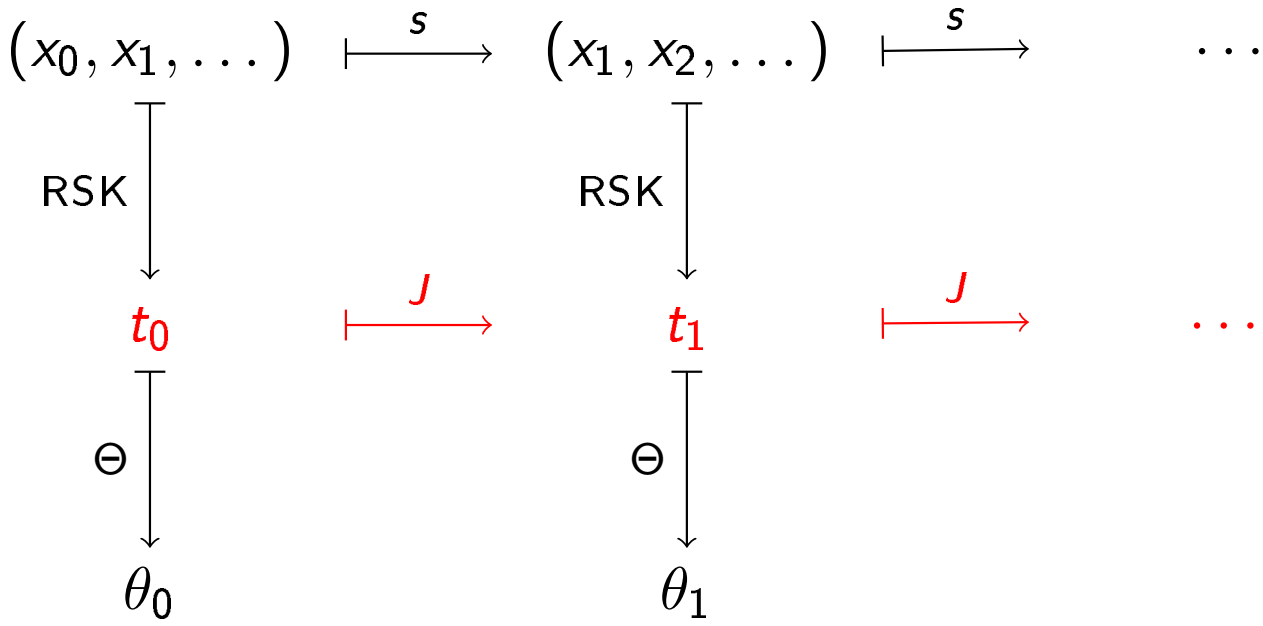


jeu de taquin dynamical system $(\Omega, \text{Plancherel}, J)$

i.i.d. shift dynamical system $\left([0,1]^{\mathbb{N}}, \prod \text{Lebesgue}, s\right)$



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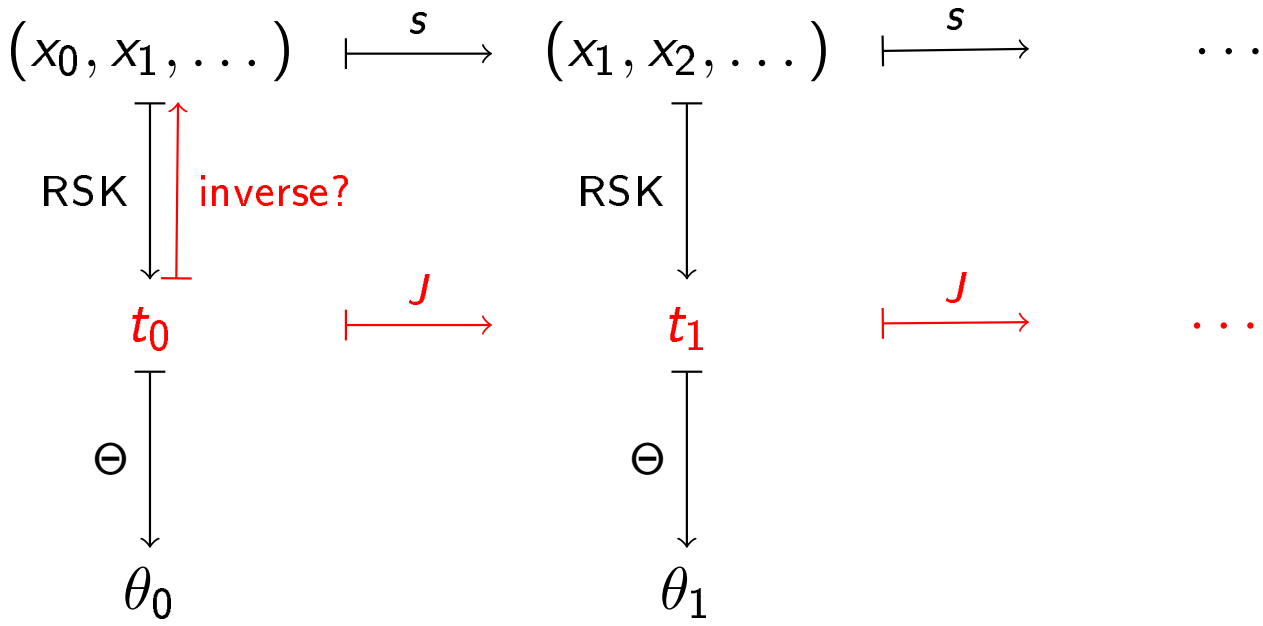

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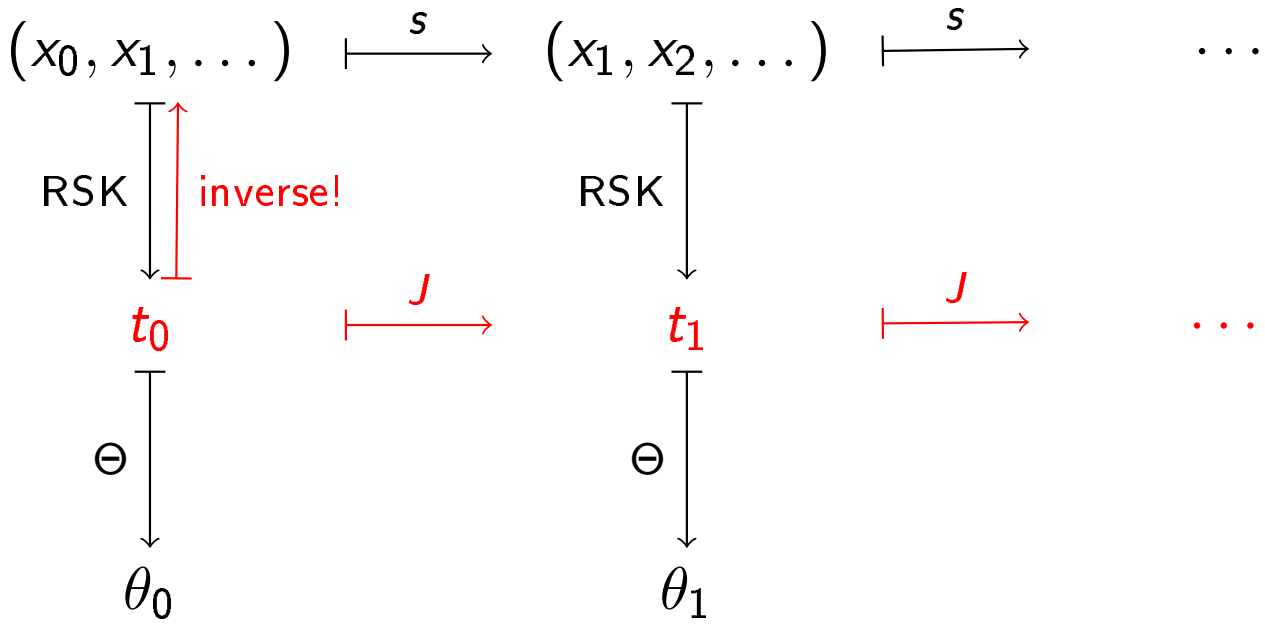

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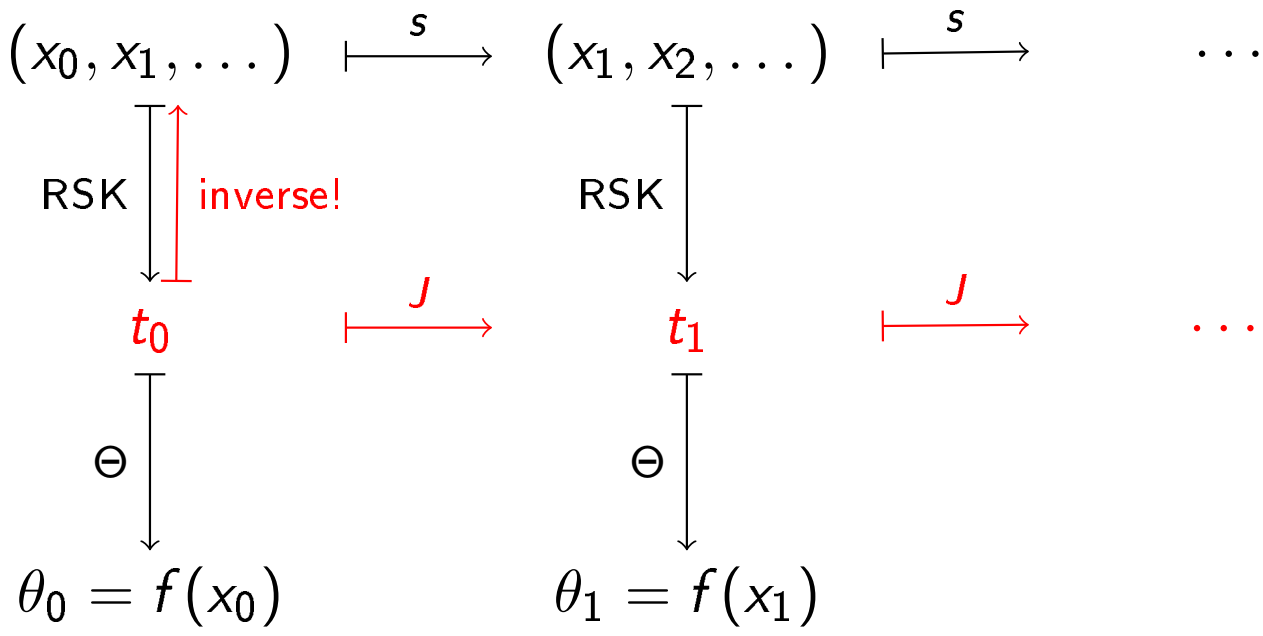

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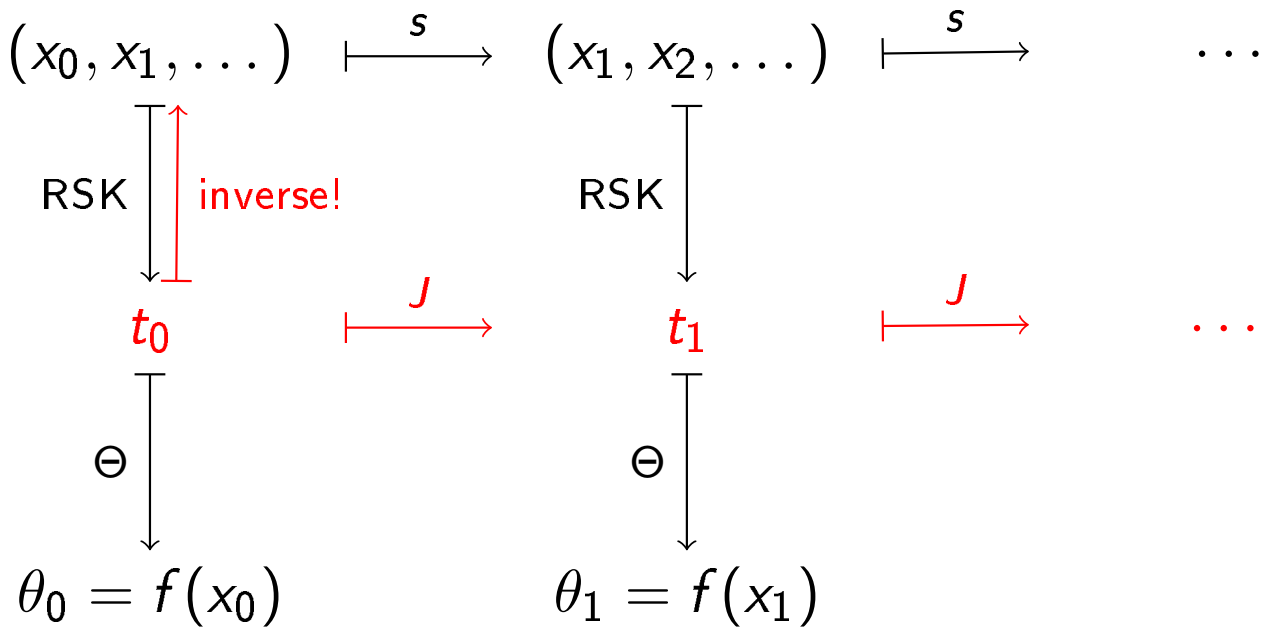


jeu de taquin dynamical system $\left(\Omega, \text{Plancherel}, J\right)$

the jeu de taquin dynamical system is isomorphic to i.i.d. shift

the inverse map is given by $x_i = f^{-1}\left(\theta_i\right)$

some consequences of the isomorphism:

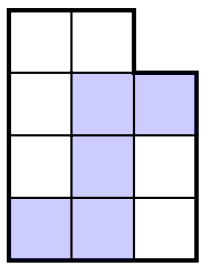
- jdt is a measure-preserving transformation,
- jdt is ergodic,
- slope angles $\theta_0, \theta_1, \ldots$ are independent random variables (put paths $\mathbf{c}(t_0), \mathbf{c}(t_1), \ldots$ are not independent),
- generalizations to other probability measures on Ω / other representations of S_∞,

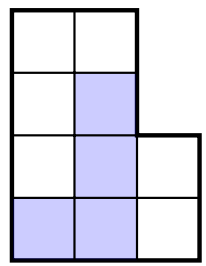
Why Θ exists and is a function of x_0?

x_0 is fixed

$x_1, x_2, \ldots$ are random, i.i.d. $U(0,1)$

$$\begin{array}{ccc} (x_0, x_1, \ldots) & \xmapsto{\quad s \quad} & (x_1, x_2, \ldots) \\ \Big\downarrow \text{RSK} & & \Big\downarrow \text{RSK} \\ \Lambda^1 \nearrow \Lambda^2 \nearrow \ldots & \xmapsto{\quad J \quad} & \overline{\Lambda}^1 \nearrow \overline{\Lambda}^2 \nearrow \ldots \end{array}$$

$\Lambda^{n+1} =$ 

$\overline{\Lambda}^{n} =$ 

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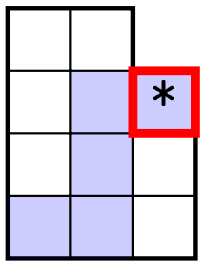
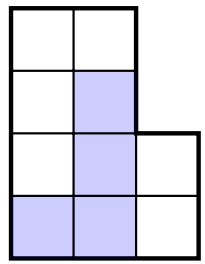
$\Lambda^{n+1} =$ 

$\overline{\Lambda}^n =$ 

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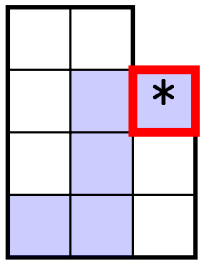
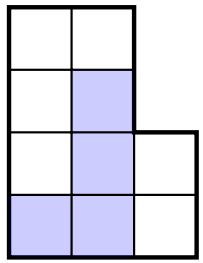
$\Lambda^{n+1} =$ 

$\overline{\Lambda}^n =$ 

$$\boxed{*} = \Lambda^{n+1} \setminus \overline{\Lambda}^n = \mathrm{RSK}(x_0, \ldots, x_n) \setminus \mathrm{RSK}(x_1, \ldots, x_n)$$

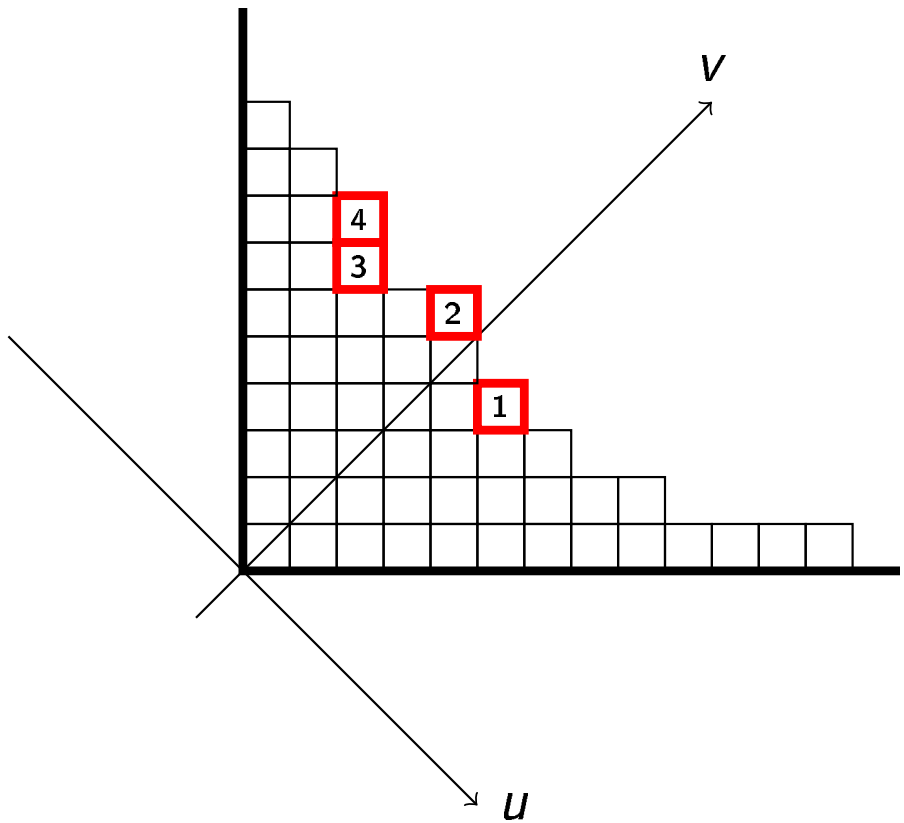
Is it true that asymptotically position of $\boxed{*}$ depends only on x_0?

instead of (for deterministic x_0) studying

$$\mathrm{RSK}(x_0, \ldots, x_n) \setminus \mathrm{RSK}(x_1, \ldots, x_n) = \boxed{*}$$

we study (for random $0 < t_1 < \cdots < t_k < 1$)

$$\mathrm{RSK}(t_1, \ldots, t_k, x_1, \ldots, x_n) \setminus \mathrm{RSK}(x_1, \ldots, x_n) = \{\boxed{1}, \ldots, \boxed{k}\}$$



Plactic Littlewood-Richarson rule

if $0 \leq x_1, \ldots, x_n \leq 1$ is a random sequence, conditioned in such a way that

$$\mathrm{RSK}(x_1, \ldots, x_n) = \lambda;$$

and $0 \leq t_1, \ldots, t_k \leq 1$ is a random sequence, conditioned in such a way that

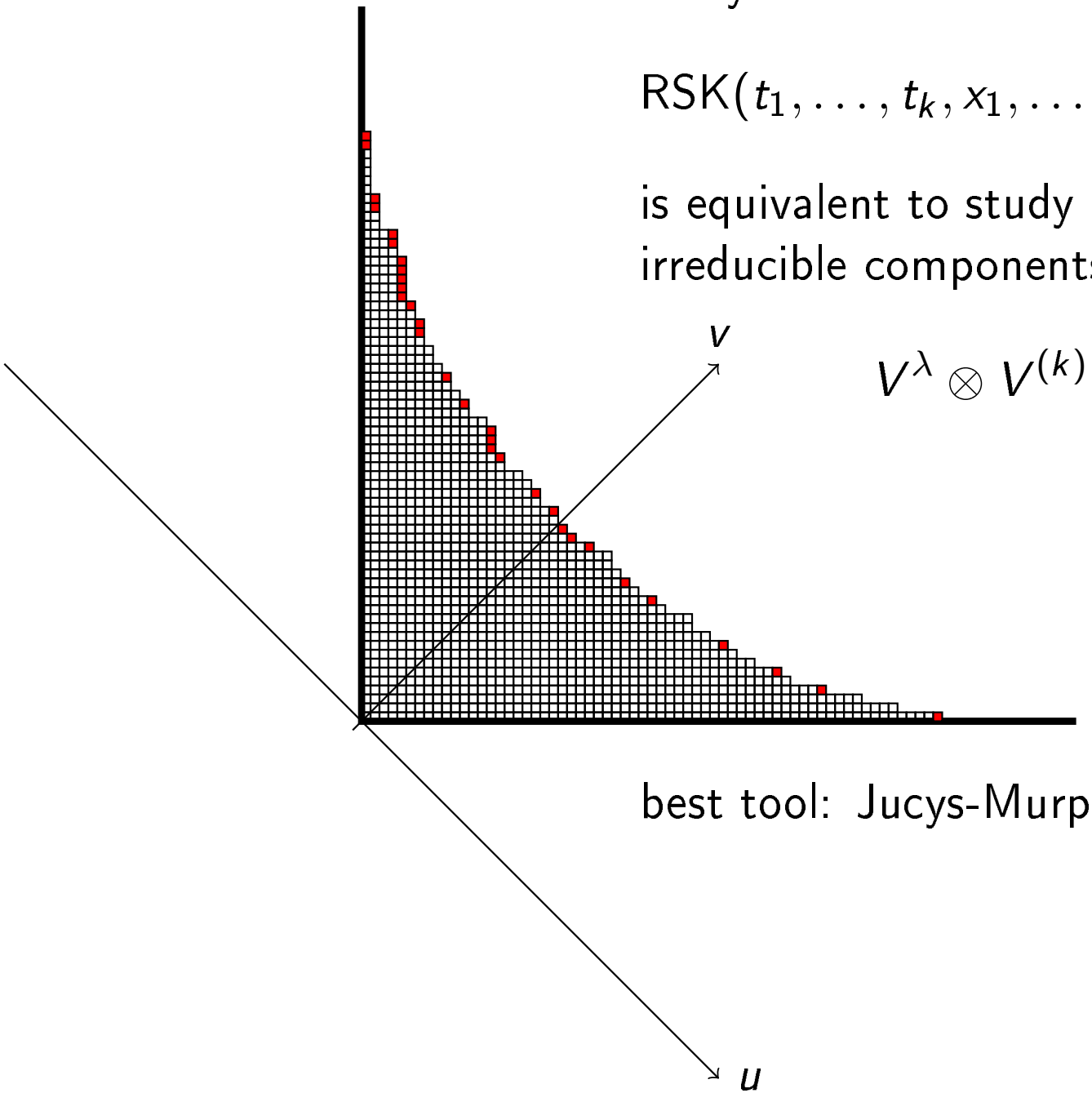
$$\mathrm{RSK}(t_1, \ldots, t_k) = \mu;$$

then the random Young diagram

$$\mathrm{RSK}(t_1, \ldots, t_k, x_1, \ldots, x_n)$$

has the same distribution as random irreducible component of

$$V^\lambda \otimes V^\mu \uparrow_{S_n \times S_k}^{S_{n+k}}$$



study of

$\mathrm{RSK}(t_1, \ldots, t_k, x_1, \ldots, x_n) \backslash \mathrm{RSK}(x_1, \ldots, x_n)$

is equivalent to study of random irreducible components of

$$V^{\lambda} \otimes V^{(k)} \uparrow_{S_n \times S_k}^{S_{n+k}}$$

best tool: Jucys-Murphy elements

Outlook: fluctuations?