

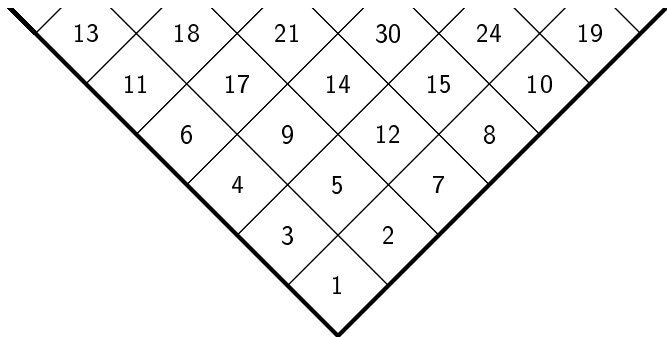
# Second class particles and random Young tableaux

joint work with Dan Romik

Piotr Śniady

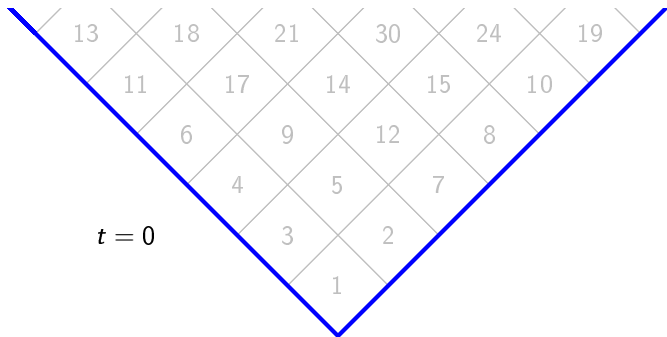
Polish Academy of Sciences  
and  
University of Wrocław

## infinite Young tableau

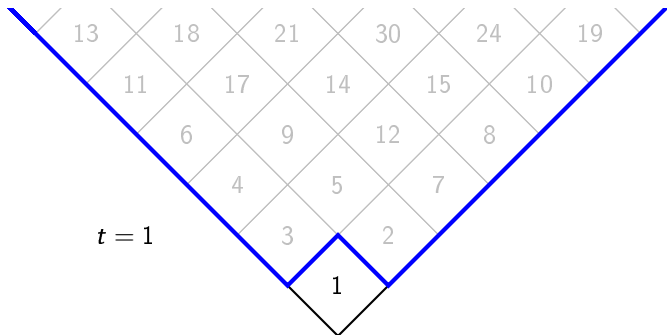


$\Omega$  — set of infinite Young tableaux

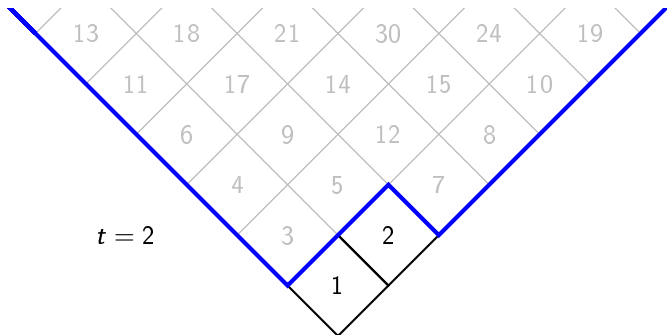
# Young tableau as a growth process



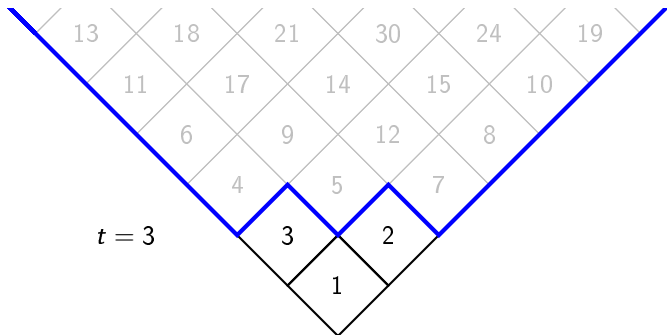
# Young tableau as a growth process



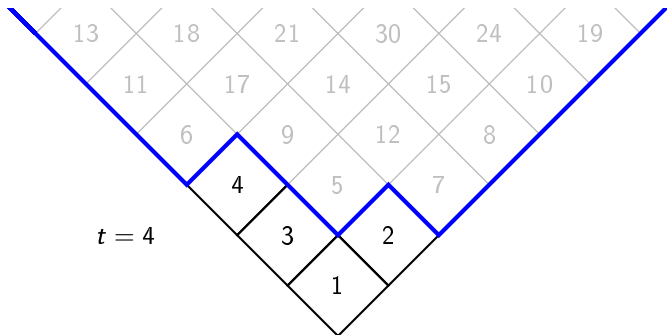
# Young tableau as a growth process



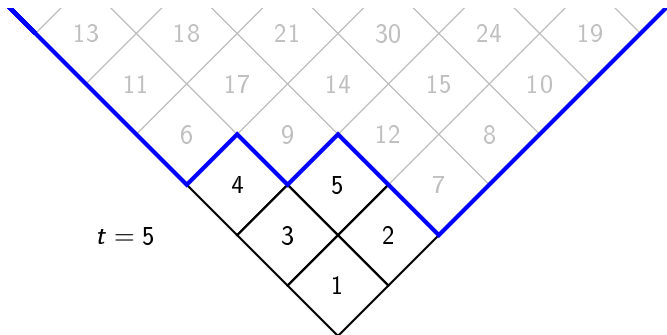
# Young tableau as a growth process



# Young tableau as a growth process

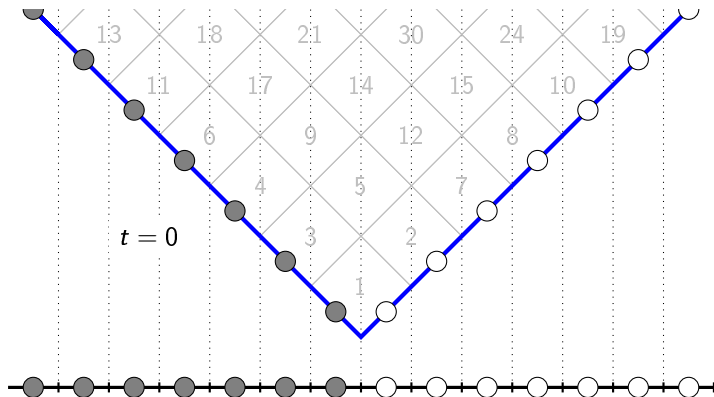


# Young tableau as a growth process

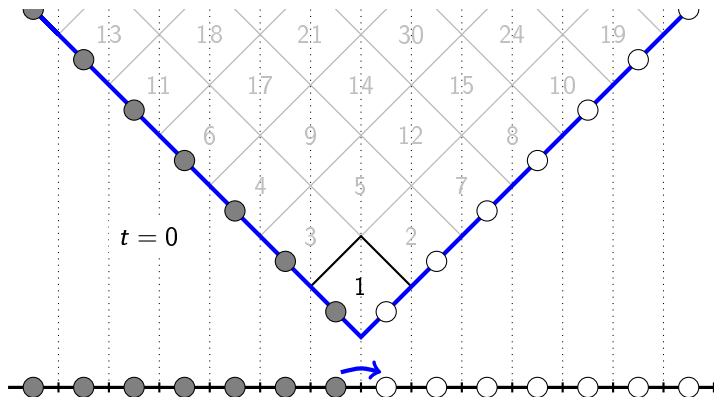




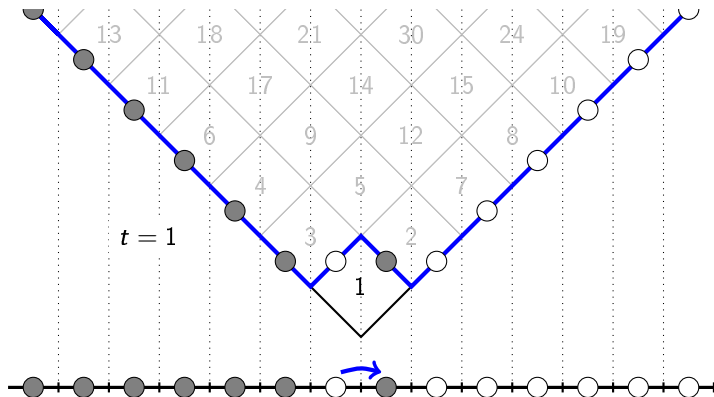
# Young tableau and dynamics of particles



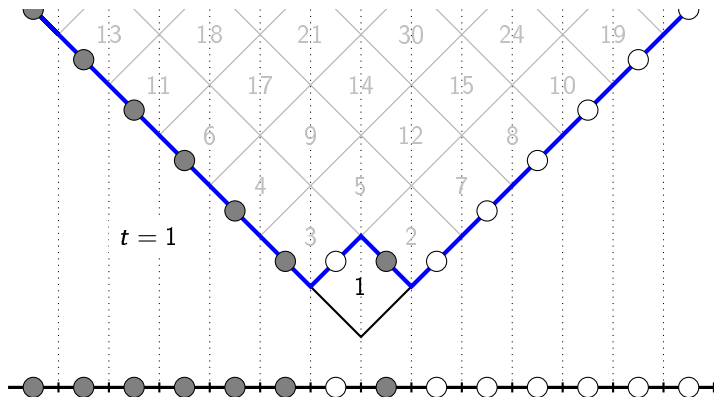
# Young tableau and dynamics of particles



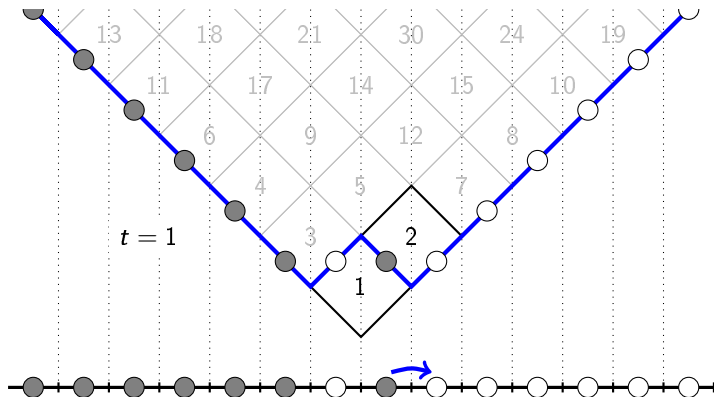
# Young tableau and dynamics of particles



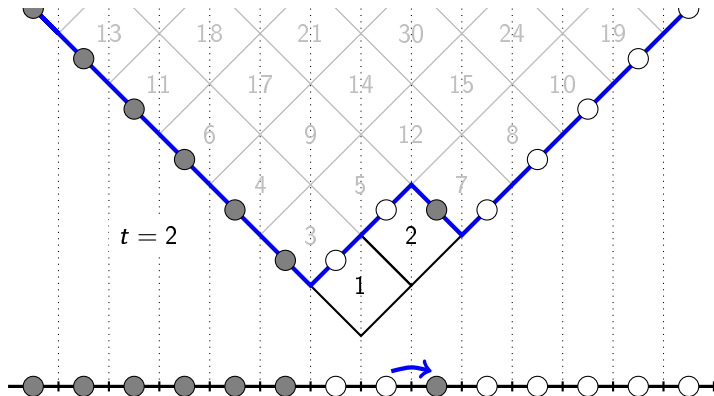
# Young tableau and dynamics of particles



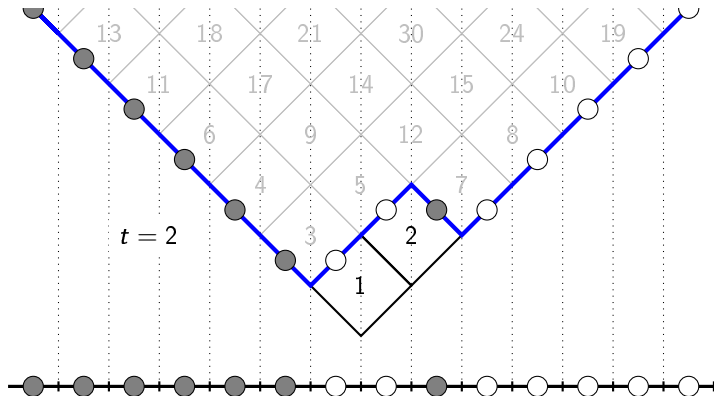
# Young tableau and dynamics of particles



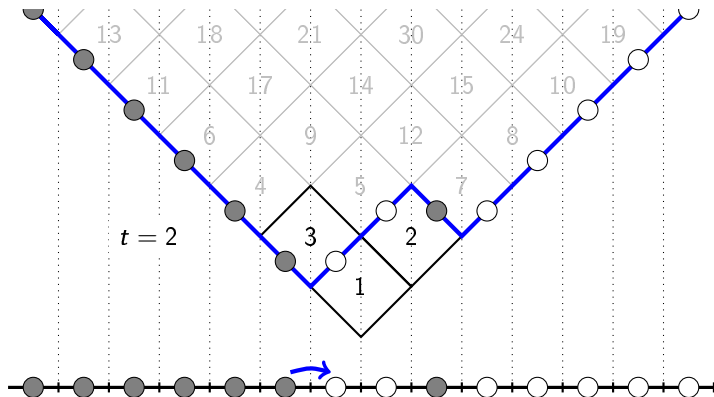
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# Young tableau and dynamics of particles

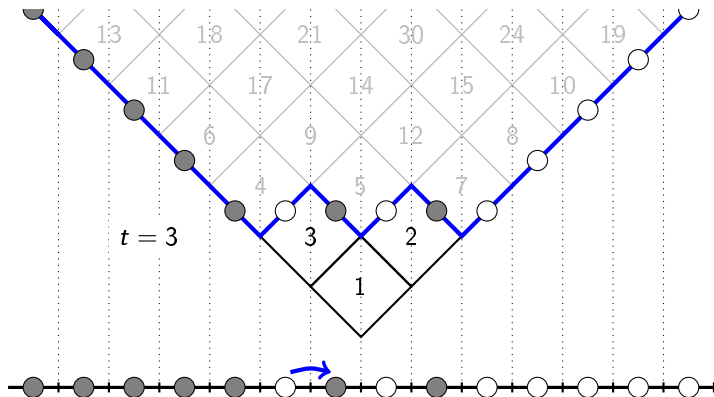


# Young tableau and dynamics of particles

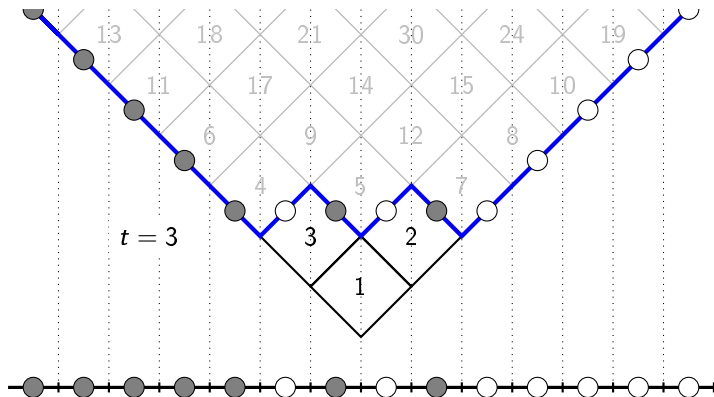




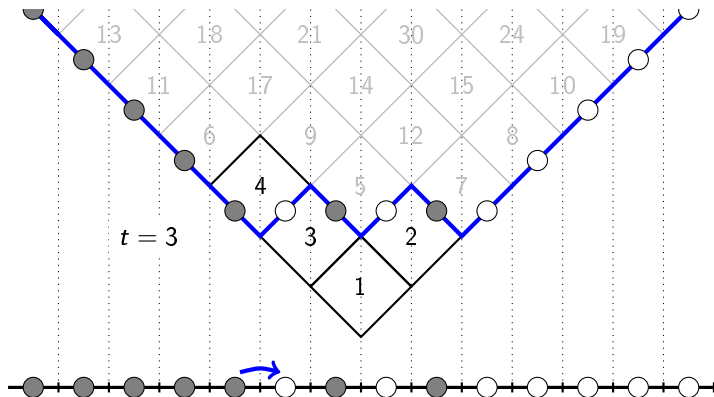
# Young tableau and dynamics of particles



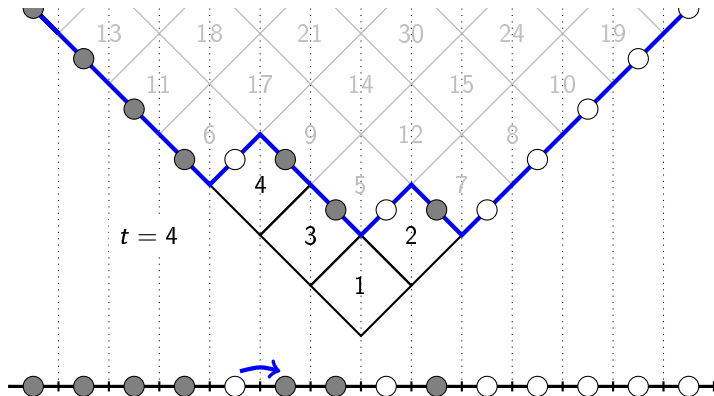
# Young tableau and dynamics of particles



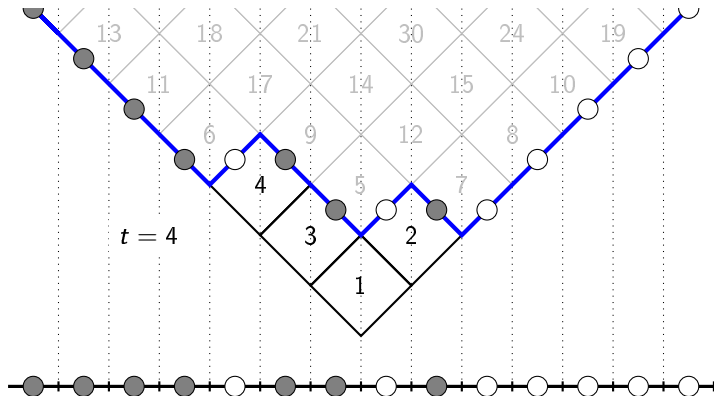
# Young tableau and dynamics of particles



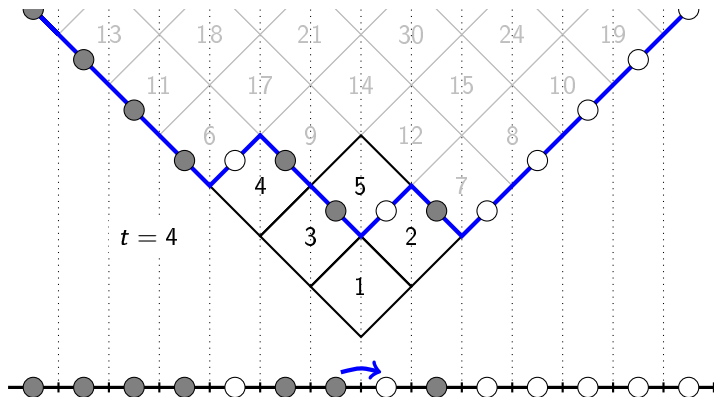
# Young tableau and dynamics of particles



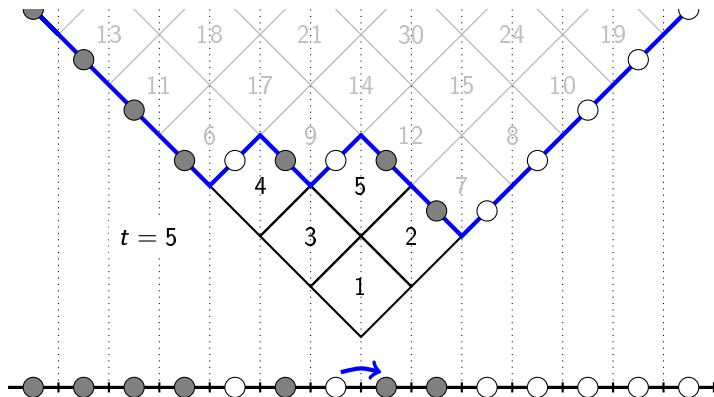
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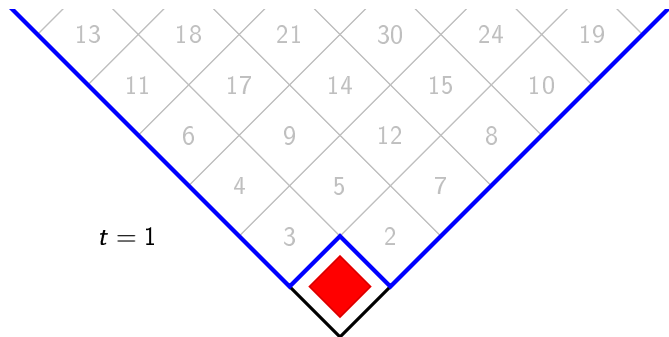
# Young tableau and dynamics of particles



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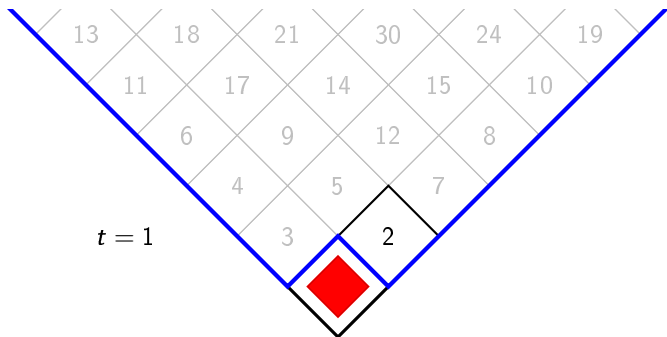
# jeu de taquin and second class particles



*jeu de taquin trajectory* — rouge path of the gap

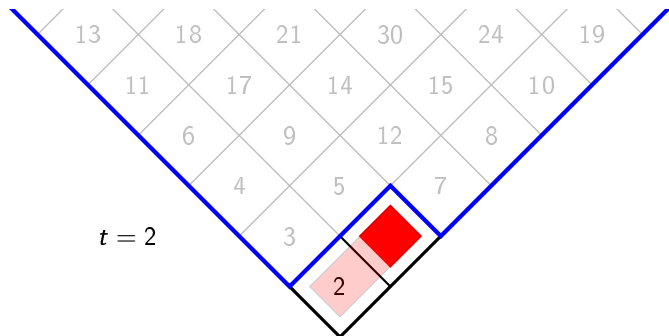


# jeu de taquin and second class particles



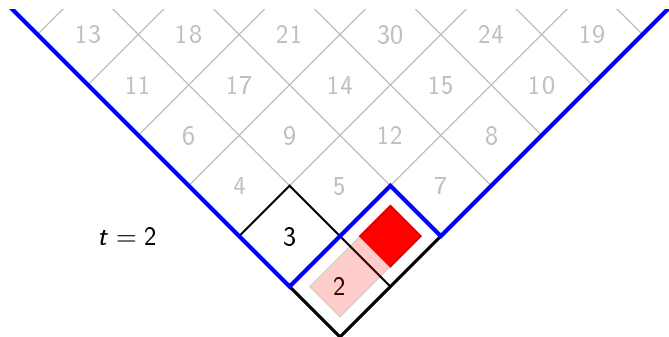
*jeu de taquin trajectory* — rouge path of the gap

## jeu de taquin and second class particles



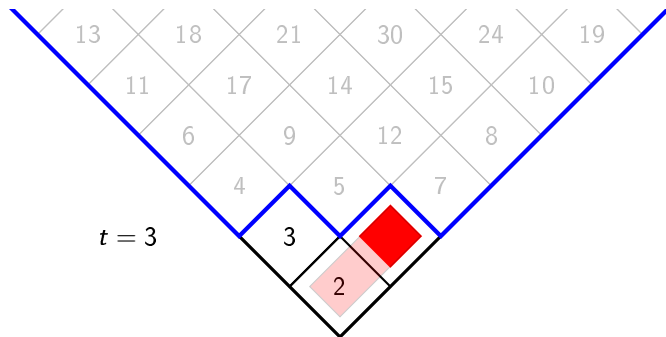
*jeu de taquin trajectory* — rouge path of the gap

## jeu de taquin and second class particles



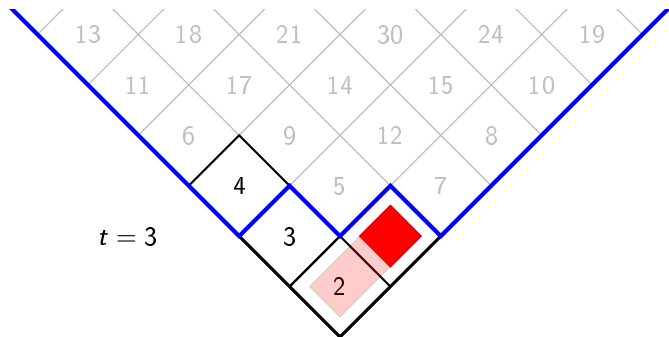
*jeu de taquin trajectory* — rouge path of the gap

## jeu de taquin and second class particles



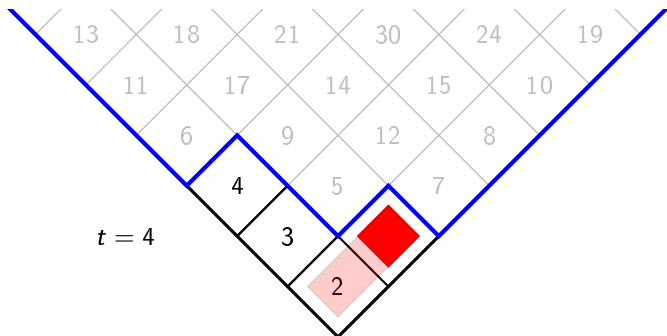
*jeu de taquin trajectory* — rouge path of the gap

## jeu de taquin and second class particles



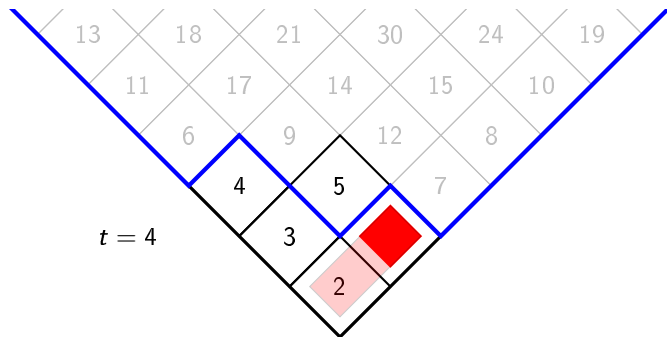
*jeu de taquin trajectory* — rouge path of the gap

## jeu de taquin and second class particles



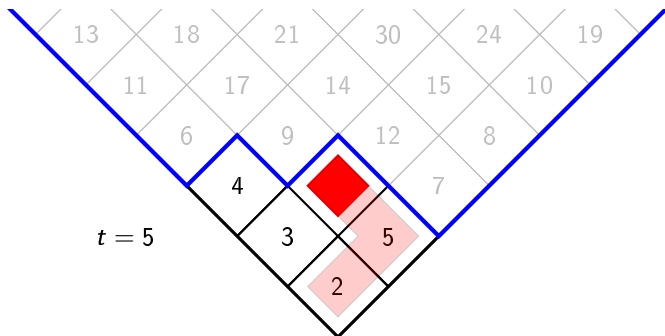
*jeu de taquin trajectory* — rouge path of the gap

## jeu de taquin and second class particles



*jeu de taquin trajectory* — rouge path of the gap

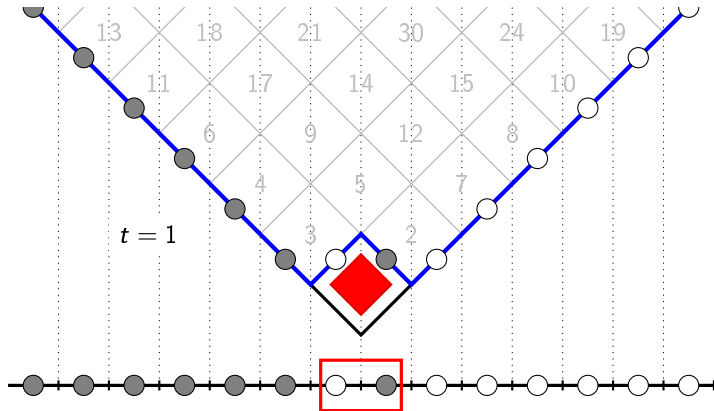
## jeu de taquin and second class particles



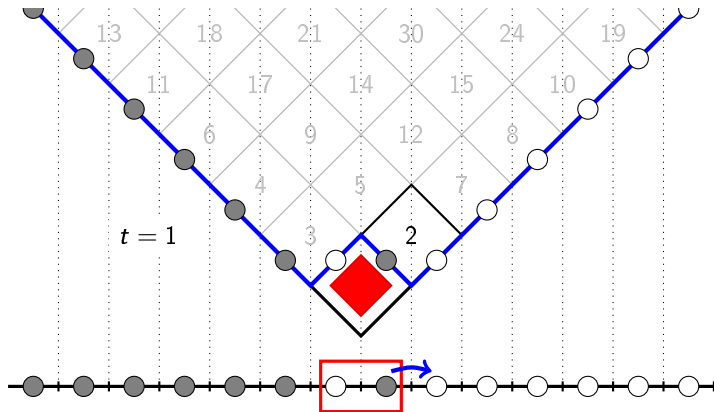
*jeu de taquin trajectory* — rouge path of the gap



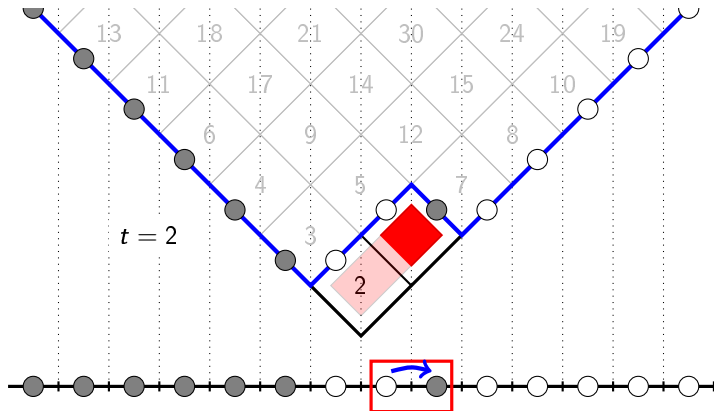
# jeu de taquin and second class particles



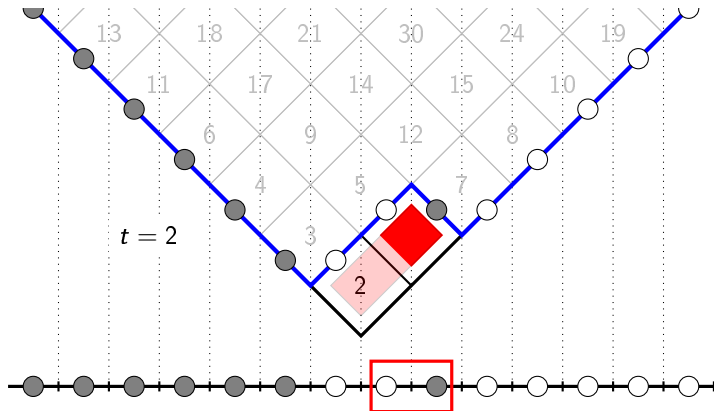
# jeu de taquin and second class particles



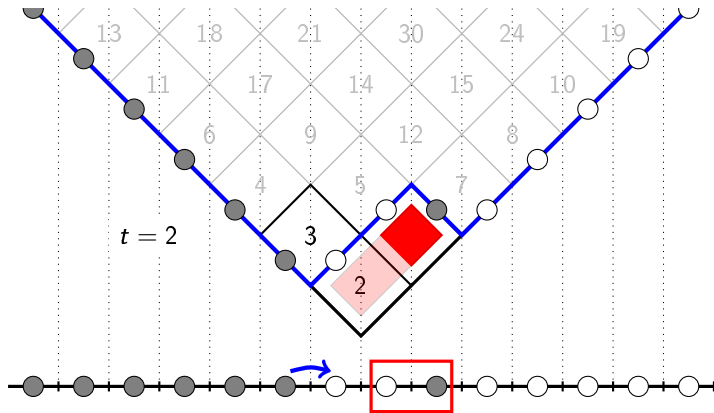
# jeu de taquin and second class particles



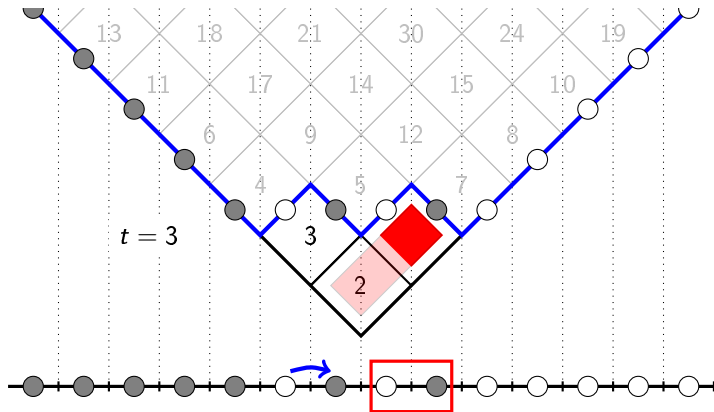
# jeu de taquin and second class particles



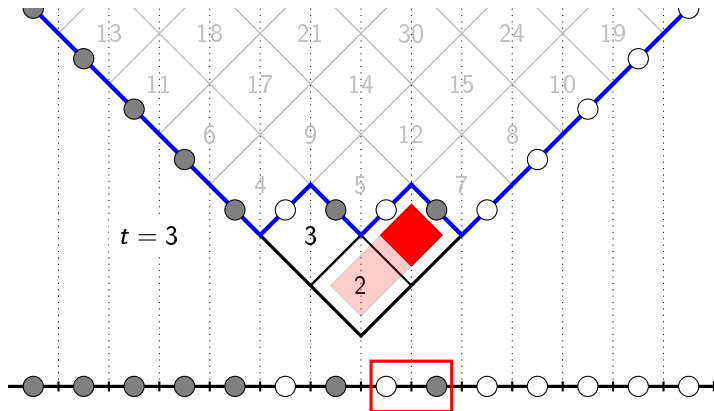
## jeu de taquin and second class particles



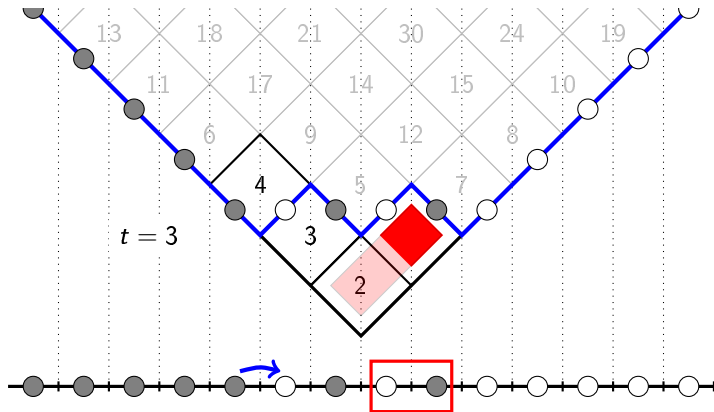
# jeu de taquin and second class particles



# jeu de taquin and second class particles

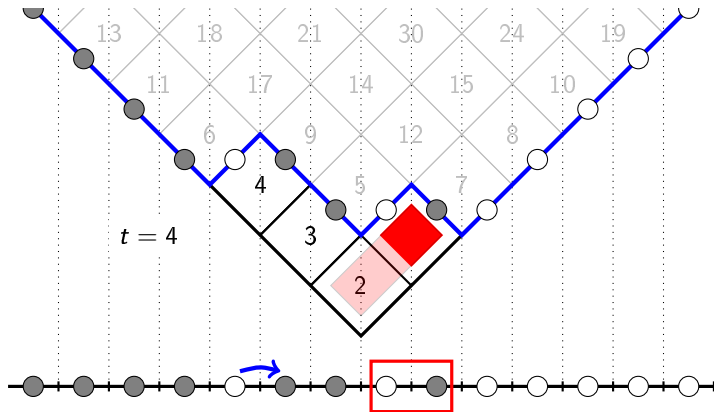


# jeu de taquin and second class particles

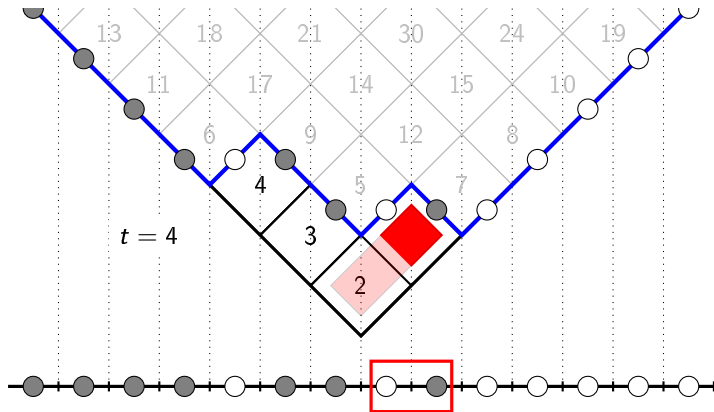




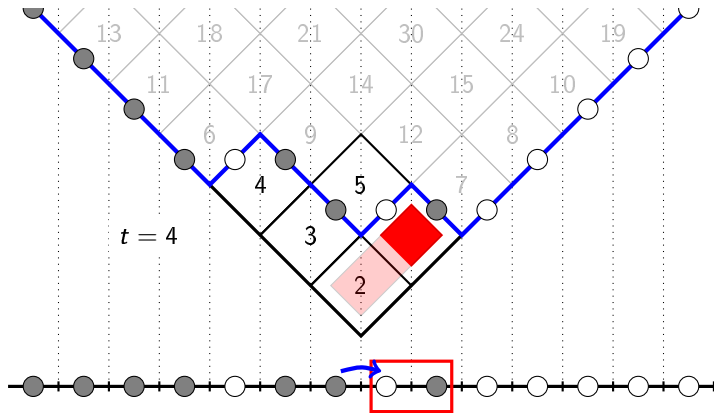
# jeu de taquin and second class particles



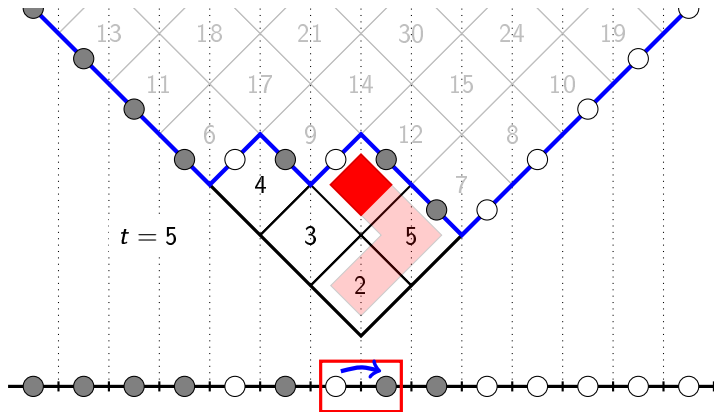
# jeu de taquin and second class particles



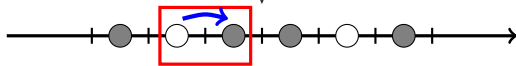
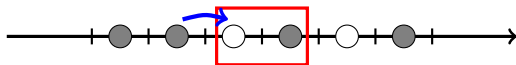
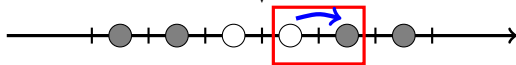
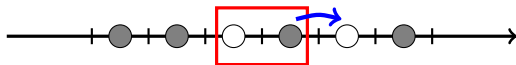
# jeu de taquin and second class particles



# jeu de taquin and second class particles

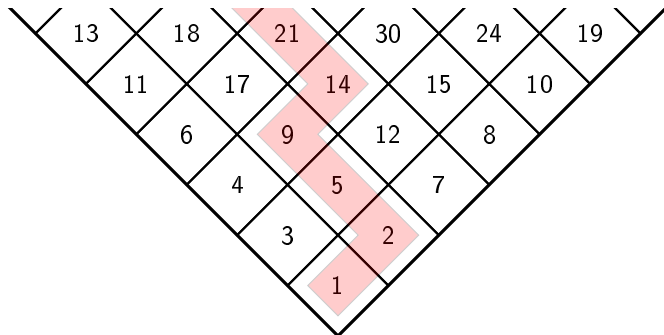


## second class particle



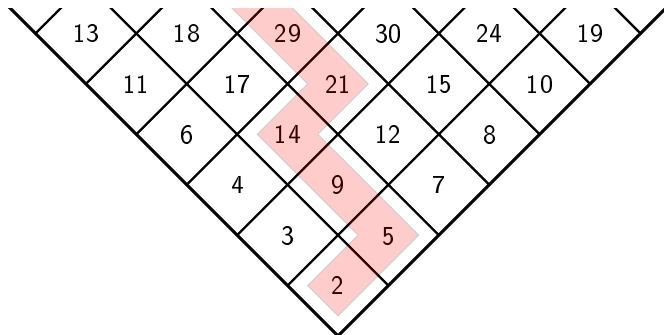
second class particle = jeu de taquin

## jeu de taquin — overview



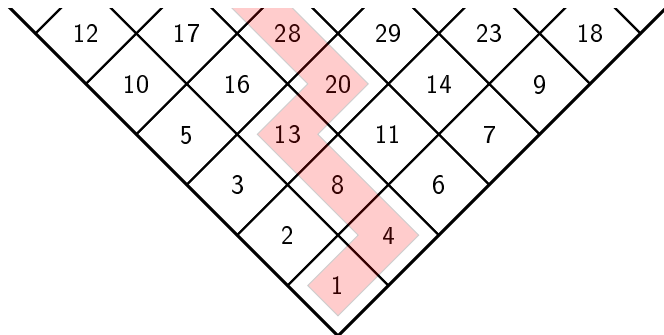
original tableau  $T$

## jeu de taquin — overview



outcome of slidings

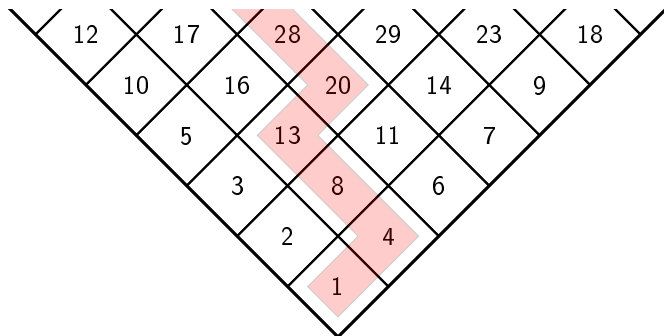
## jeu de taquin — overview



new tableau  $J(T)$



## jeu de taquin — overview

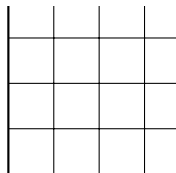


jeu de taquin applied to  $T$  gives two pieces of information:

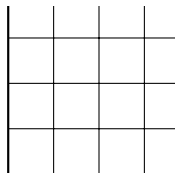
- trajectory  $\mathbf{p}(T)$  of jeu de taquin,
- the new tableau  $J(T)$ ,

# infinite Robinson-Schensted-Knuth (RSK) map

infinite word  $\xrightarrow{\text{RSK}}$  recording tableau  $\in \Omega$




insertion tableau




recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

# infinite Robinson-Schensted-Knuth (RSK) map

infinite word  $\xrightarrow{\text{RSK}}$  recording tableau  $\in \Omega$


insertion tableau

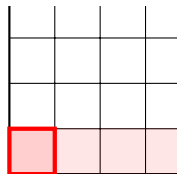

recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

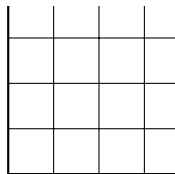
① *start from the first row,*

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insertion tableau



recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

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- ② *insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,*

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infinite word  $\xrightarrow{\text{RSK}}$  recording tableau  $\in \Omega$

F			

insertion tableau


recording tableau

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infinite word  $\xrightarrow{\text{RSK}}$  recording tableau  $\in \Omega$

F			

insertion tableau

1			

recording tableau

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④ *information about the new box into the recording tableau,*

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infinite word  $\xrightarrow{\text{RSK}}$  recording tableau  $\in \Omega$

F			

insertion tableau

1			

recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

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infinite word  $\xrightarrow{\text{RSK}}$  recording tableau  $\in \Omega$

F			

insertion tableau

1			

recording tableau

F **O** N D P X B Z U L G E A T W R S M Y V C J H Q I K

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F			

insertion tableau

1			

recording tableau

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infinite word  $\xrightarrow{\text{RSK}}$  recording tableau  $\in \Omega$

F	O		

insertion tableau

1			

recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

- ① *start from the first row,*
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# infinite Robinson-Schensted-Knuth (RSK) map

infinite word  $\xrightarrow{\text{RSK}}$  recording tableau  $\in \Omega$

F	O		

insertion tableau

1	2		

recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

- ① *start from the first row,*
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infinite word  $\xrightarrow{\text{RSK}}$  recording tableau  $\in \Omega$

F	O		

insertion tableau

1	2		

recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

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infinite word  $\xrightarrow{\text{RSK}}$  recording tableau  $\in \Omega$

F	O		

insertion tableau

1	2		

recording tableau

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infinite word  $\xrightarrow{\text{RSK}}$  recording tableau  $\in \Omega$

F	<del>O</del>		

insertion tableau

1	2		

recording tableau

F O **N** D P X B Z U L G E A T W R S M Y V C J H Q I K

- ① *start from the first row,*
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infinite word  $\xrightarrow{\text{RSK}}$  recording tableau  $\in \Omega$

F	N		

insertion tableau

1	2		

recording tableau

F O **N** D P X B Z U L G E A T W R S M Y V C J H Q I K

- ① *start from the first row,*
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infinite word  $\xrightarrow{\text{RSK}}$  recording tableau  $\in \Omega$

F	N		

insertion tableau

1	2		

recording tableau

F O **N** D P X B Z U L G E A T W R S M Y V C J H Q I K

- ① *start from the first row,*
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- ③ *insert the bumped element into the next row,*
- ④ *information about the new box into the recording tableau,*





# infinite Robinson-Schensted-Knuth (RSK) map

infinite word  $\xrightarrow{\text{RSK}}$  recording tableau  $\in \Omega$

O			
F	N		

insertion tableau

1	2		

recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

- ① *start from the first row,*
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# infinite Robinson-Schensted-Knuth (RSK) map

infinite word  $\xrightarrow{\text{RSK}}$  recording tableau  $\in \Omega$

O			
F	N		

insertion tableau

3			
1	2		

recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

- ① *start from the first row,*
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# infinite Robinson-Schensted-Knuth (RSK) map

infinite word  $\xrightarrow{\text{RSK}}$  recording tableau  $\in \Omega$

O			
F	N		

insertion tableau

3			
1	2		

recording tableau

F O **N** D P X B Z U L G E A T W R S M Y V C J H Q I K

- ① *start from the first row,*
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- ④ *information about the new box into the recording tableau,*

# infinite Robinson-Schensted-Knuth (RSK) map

infinite word  $\xrightarrow{\text{RSK}}$  recording tableau  $\in \Omega$

O			
F	N		

insertion tableau

3			
1	2		

recording tableau

F O N **D** P X B Z U L G E A T W R S M Y V C J H Q I K

- ① *start from the first row,*
- ② *insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,*
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O			
F			
D	N		

insertion tableau

4			
3			
1	2		

recording tableau

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F	L	P	W
D	G	M	R
B	E	J	Q
A	C	H	I

insertion tableau

7	11	22	24
4	10	16	17
3	9	14	15
1	2	5	6

recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

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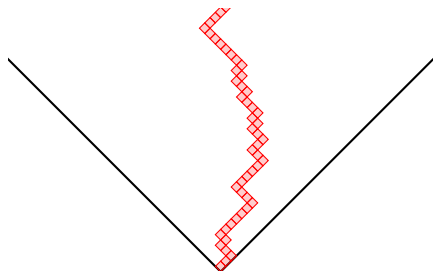
recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

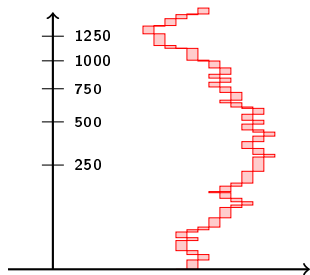
if  $X_0, X_1, \dots$  are i.i.d.  $U(0, 1)$  random variables then

$\text{RSK}(X_0, X_1, \dots)$   $\stackrel{\text{distribution}}{=} \text{Plancherel measure}$

# trajectories

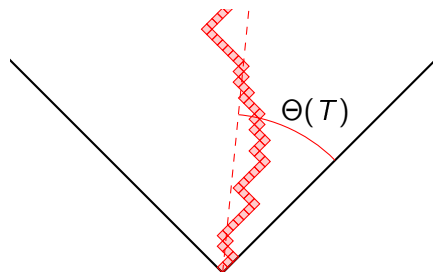


jeu de taquin



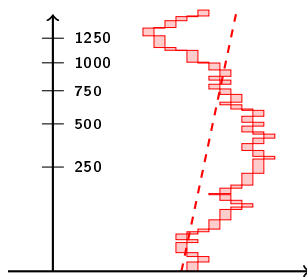
second class particle

# trajectories



jeu de taquin

$\Theta(T)$  — asymptotic angle of  
jeu de taquin

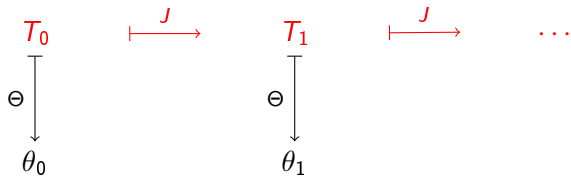


second class particle

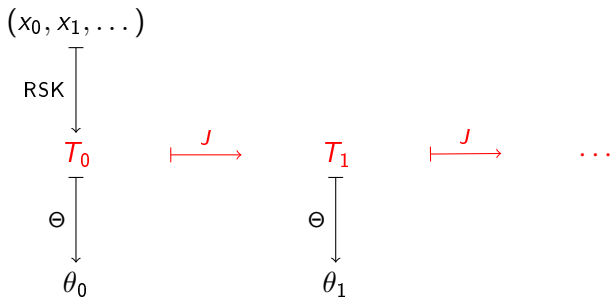
asymptotic speed of  
second class particle

$$\begin{array}{ccc} T_0 & \xrightarrow{J} & T_1 \\ \Theta \downarrow & & \\ \theta_0 & & \end{array}$$

$$\begin{array}{ccc} T_0 & \xrightarrow{J} & T_1 & \xrightarrow{J} & \dots \\ \Theta \downarrow & & \Theta \downarrow & & \\ \theta_0 & & \theta_1 & & \end{array}$$

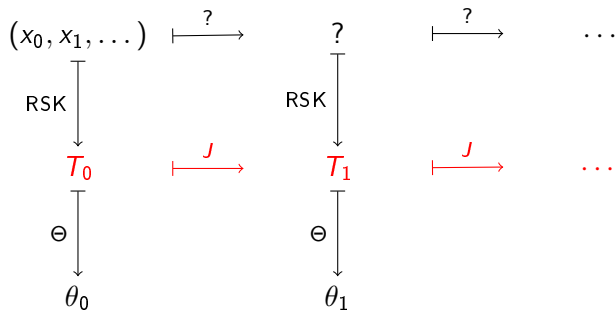


jeu de taquin dynamical system  $(\Omega, \text{Plancherel}, J)$

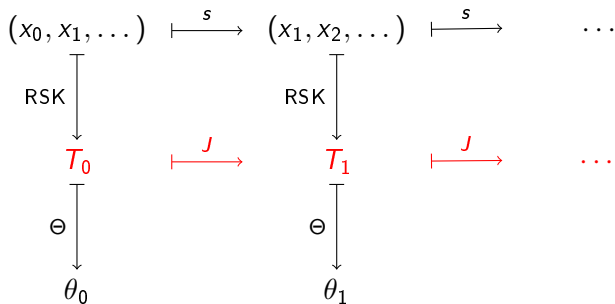


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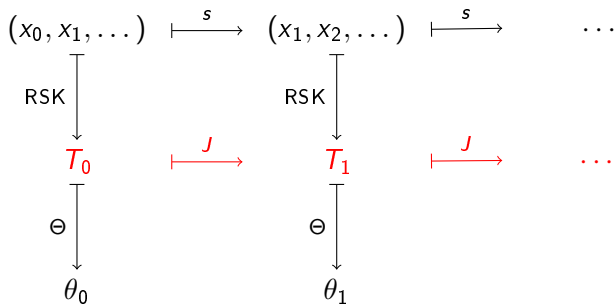


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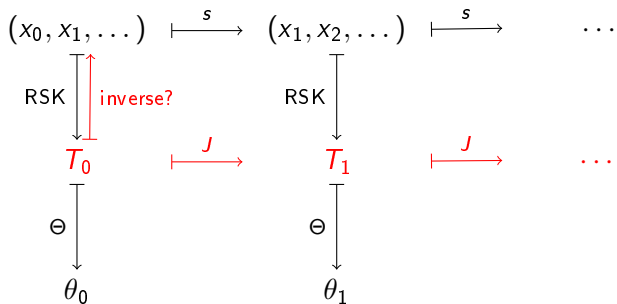
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i.i.d. shift dynamical system  $([0, 1]^{\mathbb{N}}, \prod \text{Lebesgue}, s)$



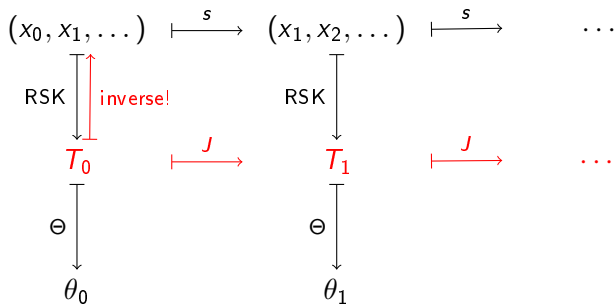
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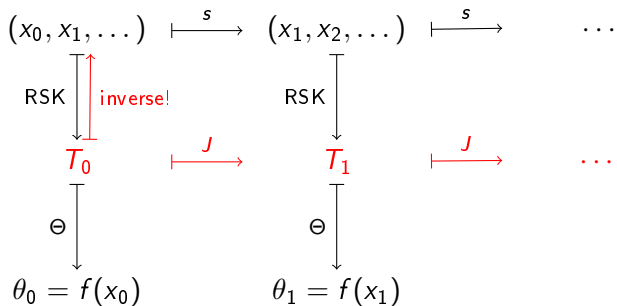
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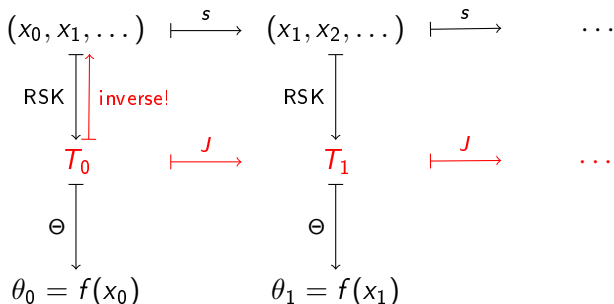
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jeu de taquin dynamical system  $(\Omega, \text{Plancherel}, J)$

the jeu de taquin dynamical system is isomorphic to i.i.d. shift

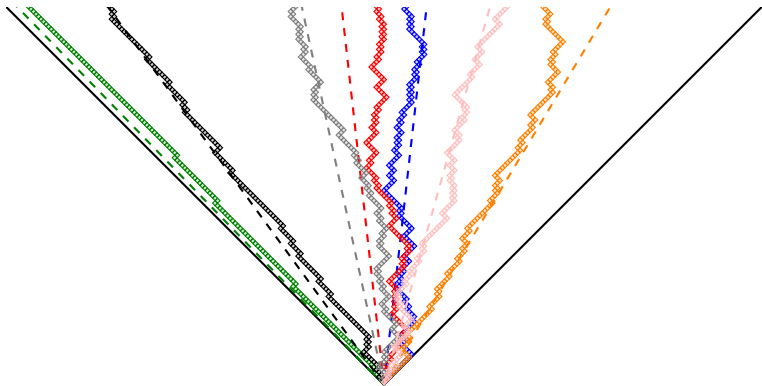
the inverse map is given by  $x_i = f^{-1}(\theta_i)$

## main results

- slope angles  $\theta_0, \theta_1, \dots$   
(and hence asymptotic speeds of second class particles!)  
exist almost surely,
- they are independent random variables with explicit  
distribution,
- RSK is an isomorphism between the dynamical system of shift  
and jeu de taquin,
- jeu de taquin is an ergodic transformation,



## open problems



Dan Romik, Piotr Śniady

Jeu de taquin dynamics on infinite Young tableaux and second class particles

[arXiv:1111.0575](https://arxiv.org/abs/1111.0575)