KONKURS MAESTRO 9
WNIOSEK
O FINANSOWANIE PROJEKTU BADAWCZEGO MAJĄCEGO NA CELU REALIZACJĘ PIONIERSKICH BADAŃ W TYM INTERDYSCYPLINARNYCH, WAŻNYCH DLA ROZWOJU NAUKI, WYKRACZAJĄCYCH POZA DOTYCHCZASOWY STAN WIEDZY, I KTÓRYCH EFEKTEM MOGĄ BYĆ ODKRYCIA NAUKOWE, REALIZOWANEGO PRZEZ DOŚWIADCZONEGO NAUKOWCA

UWAGA! Prosimy NIE WYSYŁAĆ drukowanej wersji wniosku do Narodowego Centrum Nauki.

DANE KIEROWNIKA PROJEKTU

(imię, nazwisko, tytuł lub stopień naukowy, adres zamieszkania, tel., e-mail)
prof. dr hab. Piotr Śniady
87-100 Toruń, kujawsko-pomorskie, Polska
E-mail: psniady@impan.pl

A. DANE WNIOSKODAWCY

Status wnioskodawcy:
1. Jednostka naukowa

Nazwa i adres podmiotu realizującego:
Instytut Matematyczny Polskiej Akademii Nauk
Institute of Mathematics of the Polish Academy of Sciences
ul. Śniadeckich 8, 00-656 Warszawa, mazowieckie

Siedmiocyfrowy identyfikator gminy: **146510 8**
tel: **22 5228100**
E-mail: **im@impan.pl** *, www:* **www.impan.pl**

NIP, REGON:
5250008867, 000325860

adres skrzynki podawczej ePUAP:
/IMPAN/skrytka

Status organizacyjny podmiotu:
A4. Jednostka naukowa Polskiej Akademii Nauk

Czy podmiot stanowi jednostkę zaliczaną do sektora finansów publicznych? **TAK**

Czy podmiot pozostaje pod zarządem komisarycznym lub znajduje się w toku likwidacji bądź postępowania upadłościowego?
NIE

Wnioskowane finansowanie na realizację projektu badawczego nie stanowi dla Podmiotu pomocy publicznej, o której mowa w art. 107 ust. 1 Traktatu o funkcjonowaniu Unii Europejskiej.

Kierownik podmiotu / Reprezentacja podmiotu:
prof. dr hab. czł. koresp. PAN Feliks Przytycki, Dyrektor

Czy jednostka otrzymuje dotację na działalność statutową z budżetu nauki? **TAK**

Nazwa i adres jednostki realizującej:
Instytut Matematyczny Polskiej Akademii Nauk

ul. Śniadeckich 8, 00-656 Warszawa, mazowieckie
tel: **22 5228100**
E-mail: **im@impan.pl** *, www:* **www.impan.pl**

Czy jednostka otrzymuje dotację na działalność statutową z budżetu nauki? **TAK**

B. INFORMACJE OGÓLNE

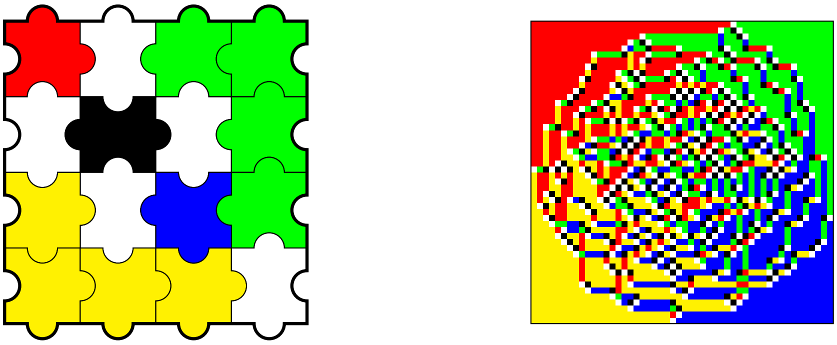
<table>
<tr><td>Tytuł projektu:</td><td colspan="3">Dynamiczna kombinatoryka asymptotyczna</td></tr>
<tr><td>Obszar badawczy:</td><td colspan="3">ST - Nauki Ścisłe i Techniczne</td></tr>
<tr><td>Numer panelu dyscyplin:</td><td colspan="3">ST1 - Nauki matematyczne</td></tr>
<tr><td>Pomocnicze określenia identyfikujące:</td><td colspan="3">ST1_14 - Kombinatoryka
ST1_13 - Rachunek prawdopodobieństwa i statystyka matematyczna
ST1_12 - Fizyka matematyczna</td></tr>
<tr><td>Planowany okres realizacji projektu (w miesiącach):</td><td>60</td><td>Liczba wykonawców projektu:</td><td>6</td></tr>
<tr><td>Słowa kluczowe:</td><td colspan="3">asymptotyczna teoria reprezentacji, kombinatoryka asymptotyczna, kombinatoryka algebraiczna, fizyka matematyczna, układy oddziałujących cząstek</td></tr>
<tr><td colspan="2" rowspan="2">Planowane nakłady w zł:</td><td>Ogółem:</td><td>Pierwszy rok realizacji:</td></tr>
<tr><td>2 608 620,00</td><td>445 620,00</td></tr>
<tr><td colspan="3">Czy projekt jest realizowany we współpracy międzynarodowej?</td><td>tak</td></tr>
<tr><td>Kraje:</td><td colspan="3">Austria , Dania , Francja , Japonia , Kanada , Meksyk , Niemcy , Republika Korei , Stany Zjednoczone Ameryki , Szwajcaria , Szwecja , Ukraina , Wielka Brytania , brak danych</td></tr>
</table>

C. **STRESZCZENIE**

Cel prowadzonych badań / hipoteza badawcza. *Co możemy powiedzieć o obiektach kombinatorycznych w granicy, gdy stają się naprawdę duże?* Przedmiotem badań projektu jest próba odpowiedzi na to bardzo ogólne pytanie dla konkretnych klasycznych przykładów pochodzących między innymi z kombinatoryki algebraicznej, teorii reprezentacji i mechaniki statystycznej. Interesujące nas pytania mają w większości charakter probabilistyczny, innymi słowy pytamy o asymptotyczne zachowanie *losowych* obiektów kombinatorycznych, na przykład: *co można powiedzieć o ich typowym zachowaniu?* Dla wielu takich modeli liczni badacze (wśród krórych znajduje się również wnioskodawca) pokazali, że losowo wybrany obiekt po przeskalowaniu koncentruje się wokół pewnego *kształtu granicznego*.

Jednym z głównych celów badawczych projektu jest udowodnienie tego typu prawa wielkich liczb, centralnego twierdzenia granicznego lub nawet bardziej wyrafinowanych wyników o charakterze probabilistycznym dla wyżej wspomnianych oraz innych probabilistycznych modeli kombinatorycznych oraz zbadanie związków z innymi, pozornie oddalonymi dziedzinami matematyki.

Szczególny akcent w projekcie poświęcony jest **zagadnieniom dynamicznym**: trajektoriom cząstek, asymptotyce algorytmów kombinatorycznych, granicy hydrodynamicznej i ogólnie zagadnieniu istnienia ***dynamicznego kształtu granicznego***.



Rysunek 1. Jeden z modeli kombinatorycznych będących przedmiotem badań projektu badawczego. Kolory odpowiadają sześciu możliwym kształtom kafelków. Gdy liczba kafelków rośnie (prawy rysunek), losowo wybrana konfiguracja z dużym prawdopodobieństwem koncentruje się wokół pewnego kształtu granicznego.

Zastosowana metoda badawcza / metodyka. Planujemy wykorzystać metody asymptotycznej teorii reprezentacji, kombinatoryki algebraicznej, algebraiczne metody teorii reprezentacji. Teoria macierzy losowych oraz wolna probabilistyka Voiculescu będą stanowić ważne źródło heurystyki.

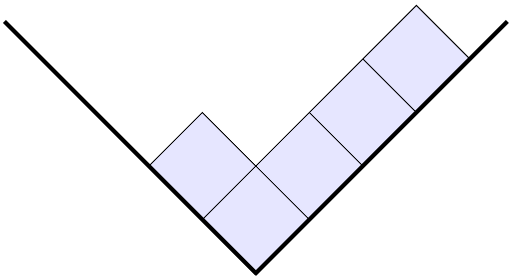
Wpływ spodziewanych rezultatów na rozwój nauki. Projekt badawczy dotyczy dziedziny, która wzbudza duże zainteresowanie społeczności matematycznej, o czym mogą świadczyć liczne zaproszone odczyty badaczy zajmujących się kształtami granicznymi na międzynarodowych kongresach matematycznych (*International Congress of Mathematicians* oraz *European Congress of Mathematics*).

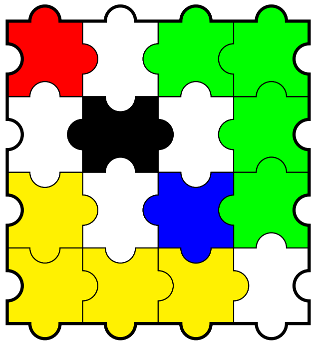
Wyniki projektu badawczego znajdą liczne zastosowania w fizyce matematycznej, zwłaszcza w konktekście modelowania zjawisk transportu w ramach fizyki statystycznej, dynamiki cząstek oraz cząstek drugiej klasy.

Określenie, w jakim stopniu i zakresie projekt obejmuje pionierskie badania naukowe, w tym interdyscyplinarne, ważne dla rozwoju nauki, wykraczające poza dotychczasowy stan wiedzy. Ważnym pionierskim elementem projektu jest *spojrzenie na duże (lub nawet nieskończone) losowe struktury kombinatoryczne z dynamicznego punktu widzenia*. Dynamika ta może oznaczać, że interesująca nas klasa obiektów jest wyposażona w jakieś naturalne odwzorowanie, którego asymptotyka ujawnia nieoczekiwane strukury wewnętrzne badanej klasy. W naturalny sposób budzi to pytania o istnienie *dynamicznego kształtu granicznego* lub o istnienie *granicy hydrodynamicznej*. Tego typu metoda stosowana była dotychczas wyłącznie w jedym przypadku (wspólna praca wnioskodawcy niniejszego wniosku z Danem Romikiem dotycząca losowych nieskończonych tablic Younga). W ramach niniejszego projektu badawczego planowane jest zastosowanie jej w znacznie szerszym spektrum przykładów.

POPULARNONAUKOWE STRESZCZENIE PROJEKTU

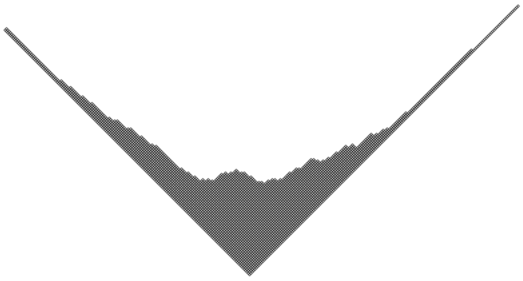
Na ile sposobów można ładnie ułożyć zadaną liczbę kwadratowych klocków w rynnie tak, aby się nie zsuwały? Na ile sposobów można wypełnić zadany obszar przy pomocy kwadratowych puzzli, z których każdy ma dwie wypustki wypukłe i dwie wypustki wklęsłe? To przykładowe pytania pochodzące z **kombinatoryki**, dziedziny matematyki zajmującej się również sudoku oraz innymi rodzajami łamigłówek.

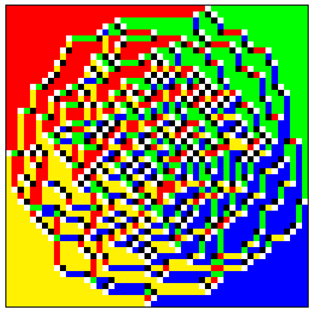




RYSUNEK 1. Przykłady obiektów kombinatorycznych będących przedmiotem badań projektu. Kolory na prawym rysunku odpowiadają sześciu możliwym rodzajom kafelków.

Takie pytania stają się oczywiście coraz trudniejsze, gdy liczba klocków rośnie. Jednocześnie jednak, gdy liczba klocków dąży do nieskończoności, można stawiać nowe, jeszcze ciekawsze pytania: *jeśli spośród wszystkich możliwych konfiguracji klocków lub puzzli wybierzemy w losowy sposób jedną, co można powiedzieć o wylosowanej w ten sposób typowej konfiguracji?* Jeśli kwadratowe klocki z pierwszej zagadki wykonać z kryształków kwarcu, powyższy problem staje się pytaniem o *typowy kształt dużej kupki piasku*. Pytania tego typu są przedmiotem badań **kombinatoryki asymptotycznej** a zarazem niniejszego projektu badawczego.





RYSUNEK 2. Bardzo duże odpowiedniki obiektów kombinatorycznych z Rysunku 1, wybrane w losowy sposób. Poszczególne kratki są tak małe, że nie zostały zaznaczone na rysunku.

Wyniki symulacji komputerowych (podobnych do tych z Rysunku 2) oraz badań teoretycznych pokazują, że w wielu takich modelach kombinatorycznych (o ile tylko ich rozmiar jest dostatecznie duży) typowa konfiguracja z prawdopodobieństwem bliskim pewności koncentruje się wokół takiego lub innego kształtu granicznego. Na przykład na diagramie z prawej strony widać „zamarznięte” jednobarwne obszary w narożnikach, podczas gdy okrągły obszar w środku wygląda jak chaotyczna „ciecz”. To zjawisko jest bardzo ciekawe z punktu widzenia fizyki matematycznej oraz fizyki statystycznej, których przedmiotem jest między innymi właśnie matematycznie ścisłe wyjaśnienie i opisanie przejść fazowych, które znamy z codziennego życia jako choćby topnienie lodu. Jednym z zadań badawczych proponowanego projektu badawczego jest matematyczne udowodnienie występowania tego typu zjawiska dla większej liczby modeli kombinatorycznych oraz zbadanie jego związków z innymi, pozornie oddalonymi dziedzinami matematyki.

Co się stanie, jeśli na szczyt kupki piasku dosypiemy więcej ziaren? Jaki kształt będzie miała lawina, jeśli usuniemy ziarna piasku leżące na samym spodzie? To przykłady **problemów dynamicznych**, którym poświęcony jest szczególny akcent w niniejszym projekcie. Odpowiedzi na takie pytania **dynamicznej kombinatoryki asymptotycznej** mogą być interesujące nie tylko dla miłośników babek z piasku oraz krzyżówkowiczów. Konfiguracje klocków w kwadratowej rynnie (fachowo zwane *partycjami*) pojawiają się w naturalny sposób w zaskakująco wielu różnych kontekstach. Jednym z nich jest *teoria reprezentacji*, która bada sposóby w jakie abstrakcyjne rodzaje symetrii mogą realizować się w konkretny sposób. Ponieważ Wszechświat jest pełny różnorakich symetrii, ta wszędobylska teoria ma bardzo liczne zastosowania. W szczególności, jeśli pewnego dnia uda się nam w pełni wykorzystać moc obliczeniową, która tkwi w *komputerach kwantowych* (lub przeciwnie: jeśli uda się nam zrozumieć ich ograniczenia), prawdopodobnie stanie się to dzięki zrozumieniu takich właśnie pozornie naiwnych pytań dotyczących babek z piasku.

JUSTIFICATION - BASIC RESEARCH

D. ANKIETA DOROBKU NAUKOWEGO KIEROWNIKA PROJEKTU

1. Imienny wykaz
(tytuł naukowy, stopień naukowy, imię, nazwisko, charakter udziału w realizacji projektu)

Tytuł zawodowy, stopień naukowy lub tytuł naukowy	Imię i nazwisko	Charakter udziału
prof. dr hab.	Piotr Śniady	Kierownik projektu

2. Ankieta dorobku naukowego

Kierownik projektu prof. dr hab. Piotr Śniady

1) Dane osobowe

Imię i nazwisko: prof. dr hab. Piotr Śniady

Typ zatrudnienia w projekcie: wynagrodzenie dodatkowe
Rodzaj stanowiska: pozostałe
Okres pobierania wynagrodzenia w projekcie (w miesiącach): 60

2) Adres zamieszkania, numer telefonu, email

Adres zamieszkania: 87-100 Toruń, kujawsko-pomorskie, Polska
Adres do korespondencji: 87-100 Toruń, kujawsko-pomorskie, Polska

E-mail: psniady@impan.pl

3) Miejsca zatrudnienia i zajmowane stanowiska

Instytut Matematyczny Polskiej Akademii Nauk (Institute of Mathematics of the Polish Academy of Sciences)
Stanowisko: profesor zwyczajny (full professor)
Adres: ul. Śniadeckich 8, 00-656 Warszawa, mazowieckie, Polska

4) Informacje o liczbie cytowań oraz indeksie H dla paneli ST (nauki ścisłe i techniczne) i NZ (nauki o życiu)

Źródło: Web of Science™ Core Collection
Łączna liczba cytowań wszystkich dotychczasowych publikacji bez autocytowań: 405
Indeks H: 12

5) Academic and Research Career (in English)

(Institution, Department/Faculty or any other Research Unit, Academic Training, Date of obtaining Academic Degree)

academic degrees

scientific degree of a professor
University of Wrocław
Department of Mathematics and Computer Science
2013

habilitation
University of Wrocław
Department of Mathematics and Computer Science
2008

doctorate
University of Wrocław
Department of Mathematics and Computer Science
2001

Master in Science, mathematics
University of Wrocław
Department of Mathematics and Computer Science
1999

Master in Science, physics

University of Wrocław
Department of Physics and Astronomy
1999

professional experience

October 2016 - now
Institute of Mathematics, Polish Academy of Sciences
Full Professor

October 2014 - September 2016
Uniwersytet im. Adama Mickiewicza w Poznaniu
Full Professor

October 2009 - September 2016
(on some periods of time: part time or on leave)
Institute of Mathematics, Polish Academy of Sciences
Associate Professor

October 2013 - September 2014
Parental leave

October 2012 - August 2014
Technische Universität München
Grant of *Deutsche Forschungsgemeinschaft*

February 2011 - August 2012
Technische Universtät München and Universität Stuttgart
Humboldt Fellowship for Experienced Researchers

September 2004 - September 2014
(on some periods of time: on leave)
University of Wrocław
Assistant Professor

August 2002 - August 2004
European Post-Doctoral Institute for Mathematical Sciences Scholarship
held at Institute des Hautes Etudes Scientifiques
(Bures-sur-Yvette, France), Ecole Normale Superieure (Paris, France), Syddansk Universitet (Odense, Denmark),
University of
Warsaw (Warsaw, Poland), Ecole Normale Superieure (Lyon, France), Universite de Franche--Comte (Besancon, France)
Postdoc

February 2002 - August 2002
University of Wroclaw
Assistant Professor

August 2001 - January 2002
Texas A&M University
Fulbright Scholarship

September 1999 - July 2000
Heidelberg Universität
DAAD Scholarship (extended studies)

August 2000 - August 2001
University of Wroclaw
Teaching Assistant

6) Publication Record for PE (Physical and Engineering Sciences) and LS (Life Sciences)

5-10 most important papers published over the period of 10 years prior to the submission of the proposal, including an index of publications in JCR-listed journals and internationally recognised research monographs (indicating five year impact factor of the journals and the number of citations of each publication excluding self-citations; 3 most important publications from the list must be attached as pdf files)

No.	Author(s)	Publication title in the original publication language	Journal (volume, pages) / monograph or monograph by many authors (editor(s), place, publisher, pages. Important note: please do not translate into English journals' and monographs' titles)	Year of publication	Current five-year impact factor	Total number of citations excluding self-citations
1	2	3	4	5	6	7
1.	Valentin Féray, Piotr Śniady	Asymptotics of characters of symmetric groups related to Stanley character formula	Annals of Mathematics (2) 173 (2011), no. 2, 887-906	2011	4,010	2
2.	Piotr Śniady	Combinatorics of asymptotic representation theory	European Congress of Mathematics, 531–545, Eur. Math. Soc., Zürich, 2013.	2013	0,000	0
3.	Cristopher Moore, Alexander Russell, Piotr Śniady	On the impossibility of a quantum sieve algorithm for graph isomorphism.	STOC'07—Proceedings of the 39th Annual ACM Symposium on Theory of Computing, 536–545, ACM, New York, 2007.	2007	0,000	11
4.	Maciej Dołęga, Valentin Féray, Piotr Śniady	Explicit combinatorial interpretation of Kerov character polynomials as numbers of permutation factorizations	Advances in Mathematics. 225 (2010), no. 1, 81–120.	2010	1,650	3
5.	Dan Romik, Piotr Śniady	Jeu de taquin dynamics on infinite Young tableaux and second class particles.	Annals of Probability 43 (2015), no. 2, 682–737	2015	2,010	2
6.	James A. Mingo, Piotr Śniady, Roland Speicher	Second order freeness and fluctuations of random matrices. II. Unitary random matrices.	Advances in Mathematics. 209 (2007), no. 1, 212–240.	2007	1,650	14
7.	Maciej Dołęga, Piotr Śniady	Asymptotics of characters of symmetric groups: structure of Kerov character polynomials.	Journal of Combinatorial Theory Series A 119 (2012), no. 6, 1174–1193.	2012	0,959	0
8.	Piotr Śniady	Robinson-Schensted-Knuth algorithm, jeu de taquin, and Kerov-Vershik measures on infinite tableaux	SIAM Journal on Discrete Mathematics 28 (2014), no. 2, 598–630	2014	0,880	0

9.	Benoît Collins, Piotr Śniady	New scaling of Itzykson-Zuber integrals.	Annales de l'Institut Henri Poincaré, Probabilités et Statistiques 43 (2007), no. 2, 139–146.	2007	1,161	9
10.	Benoît Collins, James A. Mingo, Piotr Śniady, Roland Speicher	Second order freeness and fluctuations of random matrices. III. Higher order freeness and free cumulants.	Documenta Mathematica 12 (2007), 1–70.	2007	1,000	26

7) Research projects led: both on-going and carried out in the period of 10 years prior to the submission of the proposal (only those to which one contributed as the Principal Investigator, "Kierownik" in Polish), funded under national and international funding schemes (in English)

(titles and ID numbers of projects, sources of funding, dates and places of project implementation and the list of the most important publications resulting from each project)

<table>
<tr><td colspan="5">Manually entered data</td></tr>
<tr><td rowspan="6">1.</td><td>Role in the project:</td><td colspan="3">Principal Investigator</td></tr>
<tr><td>Project title:</td><td>Dualna kombinatoryka wielomianów Jacka [Dual combinatorics of Jack polynomials]</td><td>Project ID:</td><td>2014/15/B/ST1/00064</td></tr>
<tr><td>Sources of funding:</td><td>Narodowe Centrum Nauki</td><td>Amount of funding:</td><td>543 515,00 PLN</td></tr>
<tr><td>Host Institution:</td><td colspan="3">Uniwersytet im. Adama Mickiewicza w Poznaniu</td></tr>
<tr><td>Start date:</td><td>2015-07-20</td><td>Finish date:</td><td>Project in progress</td></tr>
<tr><td>List of the most important publications:</td><td colspan="3">Agnieszka Czyżewska-Jankowska, Piotr Śniady. Jack characters and enumeration of maps. Séminaire Lotharingien de Combinatoire 78B (2017). Article #5, 12 pp. Proceedings of the 29th Conference on Formal Power Series and Algebraic Combinatorics (London)

Agnieszka Czyżewska-Jankowska, Piotr Śniady. Bijection Between Oriented Maps and Weighted Non-Oriented Maps. Electronic Journal of Combinatorics 24(3) (2017), #P3.7

Marek Bożejko, Wiktor Ejsmont, Takahiro Hasebe. Noncommutative probability of type D. Internat. J. Math. 28 (2017), no. 2, 1750010, 30 pp.

Wiktor Ejsmont. A characterization of the normal distribution by the independence of a pair of random vectors. Statist. Probab. Lett. 114 (2016), 1–5.

Wiktor Ejsmont. A characterization of symmetric stable distributions. J. Funct. Spaces 2016, Art. ID 8384767, 3 pp.</td></tr>
</table>

2.	Role in the project:	Principal Investgator		
	Project title:	Bahnen von jeu de taquin und Teilchen zweiter Klasse [Trajectories of jeu de taquin and second class particles]	Project ID:	SN 101/1-1
	Sources of funding:	Deutsche Forschungsgemeinschaft	Amount of funding:	151 800,00 EUR
	Host Institution:	Technische Universität München		
	Start date:	2012-10-01	Finish date:	2014-09-30
	List of the most important publications:	Dan Romik, Piotr Śniady, Piotr. Limit shapes of bumping routes in the Robinson-Schensted correspondence. Random Structures Algorithms 48 (2016), no. 1, 171–182 Piotr Śniady. Robinson-Schensted-Knuth algorithm, jeu de taquin, and Kerov-Vershik measures on infinite tableaux. SIAM J. Discrete Math. 28 (2014), no. 2, 598–630. Benoît Collins, Hun Hee Lee, Piotr Śniady. Dimensions of components of tensor products of representations of linear groups with applications to Beurling-Fourier algebras. Studia Math. 220 (2014), no. 3, 221–241. Motohisa Fukuda, Piotr Śniady. Partial transpose of random quantum states: exact formulas and meanders. J. Math. Phys. 54 (2013), no. 4, 042202, 23 pp. Maciej Dołęga, Valentin Féray and Piotr Śniady. Jack Polynomials and Orientability Generating Series of Maps. Séminaire Lotharingien de Combinatoire, B70j (2014), 50 pp.		

3.	Role in the project:	Principal Investigator		
	Project title:	Niekomutatywna probabilistyka, niekomutatywna analiza harmoniczna i ich zastosowania (Noncommutative probability, noncommutative harminic analysis and their applications)	Project ID:	N N201 364436
	Sources of funding:	Ministerstwo Nauki i Szkolnictwa Wyższego (Polish Ministry of Science and Higher Education)	Amount of funding:	850 000,00 PLN
	Host Institution:	Uniwersytet Wrocławski		
	Start date:	2009-05-26	Finish date:	2012-05-25
	List of the most important publications:	[the total list consists of 53 positions] Maciej Dołęga, Piotr Śniady. Asymptotics of characters of symmetric groups: Structure of Kerov character polynomials. Journal of Combinatorial Theory, Series A 119 (2012) 1174–1193 Valentin Feray, Piotr Śniady. Zonal polynomials via Stanley's coordinates and free cumulants. J. Algebra 334 (2011), 338–373. Dan Romik, Piotr Śniady. Jeu de taquin dynamics on infinite Young tableaux and second class particles. Ann. Probab. 43 (2015), no. 2, 682–737 Serban T. Belinschi, Marek Bożejko, Franz Lehner, Roland Speicher. The normal distribution is \boxplus-infinitely divisible. Advances in Mathematics 226 (2011), no. 4, 3677–3698 Dariusz Cichoń, Jan Stochel, Franciszek Hugon Szafraniec. Naimark extensions for indeterminacy in the moment problem. An example. Indiana Univ. Math. J. 59 (2010), no. 6, 1947–1970. Romuald Lenczewski. Matricial R-transform. J. Funct. Anal. 262 (2012), no. 4, 1802–1844 Marek Bożejko, Eugene Lytvynov, Janusz Wysoczański. Noncommutative Lévy processes for generalized (particularly anyon) statistics. Comm. Math. Phys. 313 (2012), no. 2, 535–569 Marek Bożejko, Eugene Lytvynov. Meixner class of non-commutative generalized stochastic processes with freely independent values II. The generating function. Comm. Math. Phys. 302 (2011), no. 2, 425–451 Rodrigo Bañuelos, Adam Osękowski. Burkholder inequalities for submartingales, Bessel processes and conformal martingales. Amer. J. Math. 135 (2013), no. 6, 1675–1698 Rodrigo Bañuelos, Adam Osękowski. Sharp inequalities for the Beurling-Ahlfors transform on radial functions. Duke Math. J. 162 (2013), no. 2, 417–434. Adam Osękowski. Sharp weak-type inequalities for Hilbert transform and Riesz projection. Israel J. Math. 192 (2012), no. 1, 429–448. Adam Osękowski. Sharp moment inequalities for differentially subordinated martingales. Studia Mathematica 201 (2010), 103–131		

4.	Role in the project:	Principal Investigator		
	Project title:	Wolna probabilistyka, niekomutatywna analiza harmoniczna i zastosowania [Free probability, noncommutative harmonic analysis and applications]	Project ID:	1 P03A 013 30
	Sources of funding:	Ministerstwo Nauki i Szkolnictwa Wyższego	Amount of funding:	630 000,00 PLN
	Host Institution:	Uniwersytet Wrocławski		
	Start date:	2006-05-05	Finish date:	2009-04-09
	List of the most important publications:	[the total list of publications consists of 39 positions] Valentin Feray, Piotr Śniady. Asymptotics of characters of symmetric groups related to Stanley character formula. Annals of Mathematics 173 (2011), 887–906 Benoit Collins, James A. Mingo, Piotr Śniady, Roland Speicher. Second order freeness and fluctuations of random matrices. III. Higher order freeness and free cumulants. Documenta Mathematica, 12 (2007), 1-70. Maciej Dołęga, Valentin Féray, Piotr Śniady. Explicit combinatorial interpretation of Kerov character polynomials as numbers of permutation factorizations. Advances in Mathematics 225 (2010) 81–120 Amarpreet Rattan, Piotr Śniady. Upper bound on the characters of the symmetric groups for balanced Young diagrams and a generalized Frobenius formula. Advances in Mathematics 218 (2008), no. 3, 673–695. Romuald Lenczewski. Decompositions of the free additive convolution. Journal of Functional Analysis 246 (2007) 330–365 Cristopher Moore; Alexander Russell; Piotr Śniady. On the impossibility of a quantum sieve algorithm for graph isomorphism. SIAM J. Comput. 39 (2010), no. 6, 2377–2396. Romuald Lenczewski. Operators related to subordination for free multiplicative convolutions, Indiana Univ. Math. J. 57 No. 3 (2008), 1055–1104 Marek Bożejko, Eugene Lytvynov. Meixner class of non-commutative generalized stochastic processes with freely independent values. I. A characterization. Comm. Math. Phys. 292 (2009), no. 1, 99–129 Waldemar Hebisch, Wojciech Młotkowski. Irreducible representations of the free product of groups. Indiana Univ. Math. J. 59 (2010), no. 1, 131–182 Piotr Śniady. Permutations without long decreasing subsequences and random matrices. Electron. J. Combin. 14 (2007), no. 1, Research Paper 11, 9 pp.		

8) Information on similar research tasks implemented or completed by projects' Principal Investigator (in English)
(project title, ID number of project, principal investigator, main research tasks, source of funding, justification of the need to fund the proposed research tasks in the light of similar tasks listed above)

This research proposal is a further continuation and extension of the project SN 101/1-1 funded by DFG (see details below). The goals of this project were achieved by the publications:

- Dan Romik, Piotr Śniady, Piotr. Limit shapes of bumping routes in the Robinson-Schensted correspondence. Random Structures Algorithms 48 (2016), no. 1, 171–182
- Piotr Śniady. Robinson-Schensted-Knuth algorithm, jeu de taquin, and Kerov-Vershik measures on infinite tableaux. SIAM J. Discrete Math. 28 (2014), no. 2, 598–630.

The scope of the current proposal is much broader.

Project title: Bahnen von jeu de taquin und Teilchen zweiter Klasse [Trajectories of jeu de taquin and second class particles]
Principal investigator: Piotr Śniady
Project ID:SN 101/1-1
Sources of funding: Deutsche Forschungsgemeinschaft
Amount of funding: 151 800,00 EUR
Host Institution:Technische Universität München
Start date: 2012-10-01
Finish date: 2014-09-30

Objectives: [copied from the original DFG application]
The goal of this research project is to nvestigate the trajectories of jeu de taquin in the finite setup as well as for various probability measures (related to the representation theory) on the set \Omega of infinite Young tableaux and to study their possible applications to harmonic analysis on the infinite symmetric group. The choice of the subject is motived by new, fascinating perspectives for random combinatorial structures and the harmonic analysis on the infinite symmetric group which became available with a very recent preprint [RomikŚniady2011]. We would like to pursue investigation of these new perspectives.
Another source of motivation comes from the theory of exclusion processes, where the trajectories of second class particles play an important role. Since we are going to study exclusion processes for which the dynamics has an additional structure given by the representation theory, more tools for studying second class particles become available and thus the subject becomes more accessible for analysis. This can give a new fresh view on a quickly developing theory of second class particles.

9) Research experience (longer research visits, placements, etc.) in Poland and abroad over the past 10 years (in English)
(country, institution, type of research stay, duration)

long stays
research grant of Deutsche Forschungsgemeinschaft
Germany
research grant funded by Deutsche Forschungsgemeinschaft,
held in Technische Universität München
August 2012- September 2014
24 months

Humboldt Fellowship for Experienced Researchers,
Germany,
research fellowship funded by Alexander von Humboldt-Foundation,
held in Technische Universtät München and Universität Stuttgart
February 2011 - August 2012
18 months

short visits
scientific visit to Kyoto University, Tokyo University and Nagoya University
Japan,
collaboration with Benoit Collins,
September 2008,
3 weeks

10) Plenary and invited lectures, presentations at renowned international conferences; in case of arts, active participation in international exhibitions, festivals, artistic events and projects in fine arts, music, theatre and film (in English)

Invited lecture *"Combinatorics of asymptotic representation theory"*
6th European Congress of Mathematics

Kraków, Poland
July 2012

Invited lecture *"Jack characters: dual combinatorics of Jack polynomials"*,
Workshop on Asymptotic Representation Theory,
Institut Henri Poincaré, Paris, France
(50 minutes)
February 2017

Invited series of lectures *"Characters, maps, free cumulants"*,
Winter school "Combinatorics and interactions",
Centre International de Rencontres Mathématiques, Luminy, France
(lectures: 3 x 90 minutes + 60 minutes exercise session)
January 2017

Invited series of lectures *"Jeu de taquin and asymptotic representation theory"*
Conference "Algebraic Combinatorics in Representation Theory",
Centre International de Rencontres Mathématiques, Luminy, France
(2 x 90 minutes)
September 2016

Invited lecture *"Dual combinatorics of Jack polynomials"*,
Final conference of the MADACA project, Domaine de Chalès, France
(60 minutes)
June 2016

Invited lecture *"Hydrodynamic limit of Robinson-Schensted algorithm"*
Séminaire Philippe Flajolet, Institut Henri Poincaré, Paris, France
(60 minutes)
December 2014

Invited lecture *"Combinatorics of asymptotic representation theory"*
Meeting of Edinburgh Mathematical Society, Aberdeen, United Kingdom
(60 minutes)
April 2014

Invited series of lectures *"Combinatorics of asymptotic representation theory",*
Seminaire Lotharingien de Combinatoire, Ellwangen, Germany
(3 x 60 minutes)
March 2013

Lecture *"Free cumulants in representation theory"*,
Conference Bialgebras in Free Probability,
Erwin Schroedinger Institute, Vienna, Austria
(50 minutes)
February 2011

Lecture *"Combinatorial interpretation of Kerov character polynomials",*
21st International Conference on Formal Power Series &Algebraic Combinatorics,
Hagenberg, Austria
(30 minutes)
June 2009

Invited lecture *"Combinatorial interpretation of Kerov character polynomials"*,
The 8th International Conference by Graduate School of Mathematics, Nagoya University "Combinatorics and Representation Theory", Nagoya University, Nagoya, Japan
(60 minutes)
September 2008

Invited lecture "*Free probability and asymptotic representation theory of symmetric groups"*,
Workshop on Free Probability, Random Matrices, and Planar Algebras,
Fields Insititue, Toronto, Canada
(50 minutes)
September 2007

Invited lecture "*Asymptotics of groups representations and free probability"*,
Conference *Noncommutative L^p-spaces, Operator spaces and Applications*,
CIRM, Marseille, France
(45 minutes)
June 2007

11) Membership in renowned scientific societies, Polish and international scientific or academic organisations (in English)
(name of the society, scientific or academic organisation, role, positions held)
Polish Mathematical Society, vicepresident

12) Membership in the scientific committees of renowned international conferences, exhibitions, festivals, artistic events (in fine arts, music, theatre and film) (in English)

25th International Conference on Formal Power Series and Algebraic Combinatorics,
Paris, France
member of the program commitee,
June 2013

24th International Conference on Formal Power Series and Algebraic Combinatorics,
Nagoya, Japan
member of the program committee
August 2012

13) Most significant research achievements (in English)
(description of up to 3 most significant research achievements in the last 10 years; in the case of research initiatives in art, also authorship of works of international impact, or works of art significant for Polish culture)

- **New asymptotic estimates for characters of the symmetric groups and their application to quantum computing.**

Valentn Féray, Piotr Śniady. Asymptotics of characters of symmetric groups related to Stanley character formula. *Ann. of Math.* (2) 173 (2011), no. 2, 887–906.

Cristopher Moore, Alexander Russell, Piotr Śniady. On the impossibility of a quantum sieve algorithm for graph isomorphism. *STOC'07—Proceedings of the 39th Annual ACM Symposium on Theory of Computing,* 536–545, ACM, New York, 2007

Cristopher Moore, Alexander Russell, Piotr Śniady. On the impossibility of a quantum sieve algorithm for graph isomorphism. *SIAM J. Comput.* 39 (2010), no. 6, 2377–2396

In a joint work with Féray we give a new combinatorial formula for the irreducible characters of the symmetric groups S_n. This formula has an advantage that it is very convenient for asymptotic problems in which the conjugacy class is fixed and the Young diagram tends to infinity. This formula also gives new strong upper bounds on the values of the characters. This part of research was published in *Annals of Mathematics.*

The main motivation for proving this kind of upper bounds comes from the theory of quantum computing: in a joint work with Moore and Russell we used them to prove that the specific version of the Hidden Subgroup Problem for the graph isomorphism **cannot** be solved quickly on a quantum computer by any version of a quantum sieve algorithm. This part of research was awarded by a talk on the celebrated STOC computer science conference.

- **Proof of Kerov positivity conjecture: explicit combinatorial interpretation for Kerov character polynomials**

Maciej Dołęga, Valentin Féray, Piotr Śniady. Explicit combinatorial interpretation of Kerov character polynomials as numbers of permutation factorizations. *Adv. Math.* 225 (2010), no. 1, 81–120.

What can we say about the irreducible characters of the symmetric groups in the limit as the size of the Young diagram tends to infinity, converging to some macroscopic shape? It was observed by Biane that the first order approximation for this question is given by *free cumulants*, quantities which originate in random matrix theory and Voiculescu's free probability theory. Later it was proved by Kerov that free cumulants can be also used for *exact* formulas for the characters, thanks to Kerov character polynomials. Kerov also conjectured that the coefficients of these polynomials are non-negative integers. Kerov conjecture was a challenging open problem and several prolific scientists, including Richard Stanley, Philippe Biane, Ian Goulden had some partial contributions.

In our work we found a constructive proof of Kerov's conjecture by proving that Kerov character polynomials are generating functions for a certain class of decorated *maps* (=graphs drawn on a twodimensional surface).

- **Asymptotic shape of jeu de taquin trajectories and ergodicity of jeu de taquin transformation**

Dan Romik, Piotr Śniady. Jeu de taquin dynamics on infinite Young tableaux and second class particles. *Ann. Probab.* 43 (2015), no. 2, 682–737

Piotr Śniady. Robinson-Schensted-Knuth algorithm, jeu de taquin, and Kerov-Vershik measures on infinite tableaux. *SIAM J. Discrete Math.* 28 (2014), no. 2, 598–630.

We investigate a *jeu de taquin transformation* (which has its origins in Schutzenberger's transformation on the set of skew semistandard Young tableaux) on the set of infinite standard Young tableaux equipped with the Plancherel probability measure or, more generally, with Kerov-Vershik measure associated to some extremal Thoma characters of the infinite symmetric group. We show that this jeu de taquin is isomorphic to a one-directional Bernoulli shift.

The key tool towards this result is a result about the dynamic limit shape of the jeu de taquin trajectory. This result is also of interest from the viewpoint of the interacting particle theory, because it can be reinterpreted as a result about the asymptotic speed of a second class particle for a TASEP system with the transition probabilities defined by the representation theory of the infinite symmetric group.

14) Most important international and prestigious Polish awards for research as well as other research activity (plenary lectures at conferences; in case of research in art, also active participation in international exhibitions, festivals, artistic events and projects in fine arts, music, theatre and film) (in English)
(type of prize/award, place and date)

selected invited talks:

6th European Congress of Mathematics
Kraków, Poland
invited speaker
2012

Séminaire Philippe Flajolet
Institute Henri Poincare, Paris
invited speaker
2013

Wojciech Pulikowski Memorial Lecture

Adam Mickiewicz University in Poznań
invited lecture
2008

Combinatorics and Representation Theory
Graduate School of Mathematics, Nagoya University, Japan
invited speaker
2008

meeting of the Edinburgh Mathematical Society
Aberdeen,
invited speaker
2013

Séminaire Lotharingien de Combinatoire
Ellwangen, Germany
invited series of lectures
2013

Winter school “Combinatorics and interactions”
Centre International de Rencontres Mathématiques, Luminy, France
invited series of lectures
2017

conference "Algebraic Combinatorics in Representation Theory",
Centre International de Rencontres Mathématiques, Luminy, France
invited series of lectures
2016

selected awards:

award of Narodowe Centrum Nauki
2015

award of Institute of Mathematics (Polish Academy of Sciences) for outstanding mathematical achievements
[national award for mathematicians below 45 years of age]
2017

Wacław Sierpiński Memorial Award
(Division III of Polish Academy of Sciences)
2011

Polish Prime Minister Award for the habilitation dissertation
2009

award of Polityka Foundation
2008

Scholarship for Excellent Young Scientists
Polish Ministry of Science and Higher Education
2008-2011

PLAN BADAŃ

Lp.	Nazwa zadania badawczego w języku polskim	Podmiot realizujący zadanie
1.	Symulacje numeryczne, komputerowa eksploracja wyników symulacji, statystyczne testowanie hipotez dotyczących (nie)gaussowskości trajektorii jeu de taquin	Instytut Matematyczny Polskiej Akademii Nauk
2.	Komputerowe obliczenia symboliczne, komputerowo wspomagana eksploracja danych w poszukiwaniu regularności w danych	Instytut Matematyczny Polskiej Akademii Nauk
3.	Udowodnienie istnienia deterministycznej granicy hydrodynamicznej dla "insertion tableau" w algorytmie Robinsona-Schensteda-Knutha	Instytut Matematyczny Polskiej Akademii Nauk
4.	Znalezienie operacji na reprezentacjach grup permutacji S_n, która śledziłaby ewolucję trajektorii jeu de taquin	Instytut Matematyczny Polskiej Akademii Nauk
5.	Udowodnienie istnienia dynamicznych kształtów granicznych dla trajektorii jeu de taquin dla losowych tableau Younga wypełniających zadany kształt	Instytut Matematyczny Polskiej Akademii Nauk
6.	Zbadanie trajektorii cząstek drugiego rodzaju dla modelu TASEP z jednorodnym rozkładem prawdopodobieństwa na historiach	Instytut Matematyczny Polskiej Akademii Nauk
7.	Udowodnienie istnienia granicy hydrodynamicznej dla dyskretnego modelu opróżniania zbiornika z piaskiem (iterowanie odwzorowania jeu de taquin)	Instytut Matematyczny Polskiej Akademii Nauk
8.	Zbadanie istnienia (dynamicznych) kształtów granicznych dla innych modeli	Instytut Matematyczny Polskiej Akademii Nauk

UWAGA: Zadaniem badawczym nie jest np. zakup aparatury, udział w konferencji, przygotowanie publikacji itp. Zadania badawcze muszą zawierać tożsamą treść w języku polskim i angielskim.

F. KOSZTORYS

Pozycja	Rok 2018	Rok 2019	Rok 2020	Rok 2021	Rok 2022	Rok 2023	Razem
1	2	3	4	5	6	7	8
Koszty bezpośrednie realizacji projektu, w tym:	330 620	424 500	382 500	364 500	325 500	48 000	1 875 620
- wynagrodzenia wraz z pochodnymi i stypendia naukowe	177 000	324 000	318 000	300 000	273 000	48 000	1 440 000
- koszty aparatury naukowo-badawczej, urządzeń i oprogramowania	43 120	0	0	0	0	0	43 120
- inne koszty bezpośrednie	110 500	100 500	64 500	64 500	52 500	0	392 500
Koszty pośrednie	115 000	169 800	153 000	145 800	130 200	19 200	733 000
Koszty realizacji projektu ogółem	445 620	594 300	535 500	510 300	455 700	67 200	2 608 620

Kalkulacja i uzasadnienie poszczególnych pozycji kosztorysu

1) Investigator / Staff Costs

a) Number of Investigators: 6

The number of individuals to constitute the basis for the calculation of additional remuneration budget: 1

b) Investigators: nature of their contribution in the project and a justification of investigator costs

No.	Full name / Nature of contribution in the project / Type and character of position	Employing entity	Project-related remuneration period (months)	Total salary cost on grant (PLN)
1.	prof. dr hab. Piotr Śniady Principal Investigator Position: no full-time position, other	Institute of Mathematics of the Polish Academy of Sciences	60	360 000
	Scope of work within individual project tasks:	All tasks, coordination of work, supervision of PostDoc, supervision of students and PhD students		
2.	Co-investigator 1 Co-Investigator Position: full-time position, post-doc type	Institute of Mathematics of the Polish Academy of Sciences	60	600 000
	Scope of work within individual project tasks:	Tasks 3-8 with a special emphasis on Task 8.		
3.	Co-investigator 2 Co-Investigator Position: scholarship position, PhD student type	Institute of Mathematics of the Polish Academy of Sciences	48	216 000
	Scope of work within individual project tasks:	Research tasks 3-7, with more emphasis on representation theory and combinatorics.		
4.	Co-investigator 3 Co-Investigator Position: scholarship position, PhD student type	Institute of Mathematics of the Polish Academy of Sciences	48	216 000
	Scope of work within individual project tasks:	Research tasks 3-7, with more emphasis on probability theory.		
5.	Co-investigator 4 Co-Investigator Position: scholarship position, student	Institute of Mathematics of the Polish Academy of Sciences	24	24 000
	Scope of work within individual project tasks:	Research tasks 1-2: programming of numerical simulations and symbolic calculations (SageMath, Python)		
6.	Co-investigator 5 Co-Investigator Position: scholarship position, student	Institute of Mathematics of the Polish Academy of Sciences	24	24 000
	Scope of work within individual project tasks:	Research tasks 1-2: programming of numerical simulations and symbolic calculations (SageMath, Python)		
			Total:	1 440 000

2) List of equipment to be purchased and/or built

No.	Nazwa aparatury	Purchasing entity	Year of purchase	Unit Cost (PLN)	Amount	Cost (PLN)	Contribution of the NCN (PLN)
1.	ultramobilny laptop	Institute of Mathematics of the Polish Academy of Sciences	2018	8 000	2	16 000	16 000
	Justification of purchase:	This research proposal is planned to be performed with several domestic partners which reside in the cities of Warsaw, Toruń, Poznań, Wrocław, Kraków and Gdańsk. For this reason the mobility of the members of the team will be very high and ultramobile laptops will be essential for the work comfort while travelling. The same applies to less frequent foreign visits.					
2.	laptop	Institute of Mathematics of the Polish Academy of Sciences	2018	4 000	2	8 000	8 000
	Justification of purchase:	This research proposal is planned to be performed with several domestic partners which reside in the cities of Warsaw, Toruń, Poznań, Kraków and Gdańsk. We also plan to make several scientific visits and conference trips. In order to work efficiently, the members of the team need laptops.					
3.	tablet z aktywnym digitizerem	Institute of Mathematics of the Polish Academy of Sciences	2018	4 780	4	19 120	19 120
	Justification of purchase:	Since the research is performed in collaboration with foreign partners we need an efficient way of communicating and teleconferencing. Mathematical research involves a heavy number of sketches, diagrams, etc. which for informal discussions look best when drawn by hand. This naturally calls for the usage of digital ink. Leading brand tablet (12.9 inch): 4300 PLN. Active digital pen (sold separately): 480 PLN					
					Total:	43 120,00	43 120,00

3) Other costs justification (in English)
(Please list and justify type of costs, estimated costs in accordance with the research plan)

Materials
(expendable goods for direct use in the project)
Cost: 20 000,00
We plan to buy around 10 books; the average price is 90 EUR = 382 PLN; the total is 10 x 382 = 3 820 PLN.

The annual cost of materials and small equipment (paper, toner for printers, USB memory sticks, external data storage, remote presentation devices, etc) is 3236 PLN; the total is 5 x 3236 = 16 180 PLN.

Conferences and business trips
(by members of the research team)
Cost: 247 500,00

The average cost of a domestic conference is around 1500 PLN; the members of the team will attend on average 4 conferences per year; thus the total cost of domestic conferences is 5 x 4 x 1500 = 30 000 PLN.

The average cost of a short domestic trip is around 250 PLN; the members of the team will make on average 30 such trips per year; thus the total cost of short domestic trips is 5 x 30 x 250 = 37 500 PLN.

The average cost of a European conference is 6000 PLN, during the whole project we plan a total of 20 such trips which amounts to 20 x 600 = 120 000 PLN.

The average cost of a conference outside of Europe is 10000 PLN, during the project we plan 6 such trips which amounts to 60 000 PLN.

Legal note: above, whenever we speak about a *'conference'* we mean a conference, a scientific workshop, a scientific school, a collaboration, etc.

Visits and consultations
(travel expenses / travel expenses by external collaborators and/or consultants and costs of meetings)
Cost: 120 000,00

During the research project we plan a total of 20 visits of our foreign and domestic collaborators. The average cost of the visit (including the travel, housing and optionally the conference fee) is 6000 PLN. The total cost is 20 x 6000 = 120 000 PLN.

Other costs
(such as do not fit into other categories, including the cost of disseminating the results)
Cost: 5 000,00
We plan to spend 5000 PLN for publication fees including open-access fees as well as voluntary publication fees, as well as on editing services.

4) Investigators' qualifications
(Qualifications required from investigators involved in the project)

- **Coinvestigator 2 (Post-doc)** needs to have a PhD in mathematics, computer science, or physics. An ideal candidate should have the knowledge of algebraic combinatorics and representation theory. Experience in probability theory would be advantagious. Alternatively, the candidate should have the experience in theoretical physics or mathematical physics.
- **Coinvestgators 3 and 4 (PhD students)** needs to have a MSc in mathematics, computer science or physics.
- **Coinvestigators 5 and 6 (students)** needs to be students of mathematics, computer science, physics or related studies. They need programming skills in languages such as Python and/or Sage and basic knowledge of probability and statistics.

G. OŚWIADCZENIA

G1. OŚWIADCZENIA KIEROWNIKA PROJEKTU prof. dr hab. Piotr Śniady

1. Oświadczam, że zadania badawcze, objęte niniejszym wnioskiem, nie są i nie były finansowane z Narodowego Centrum Nauki, jak również z innego źródła.
   Oświadczam, że równocześnie nie ubiegam / ~~ubiegam~~ się o finansowanie tych zadań z innych źródeł:
2. Oświadczam, że w przypadku uzyskania finansowania na zadania objęte wnioskiem z innego źródła niezwłocznie poinformuję o tym fakcie Narodowe Centrum Nauki, i:
   a. powiadomię osobę upoważnioną do reprezentowania podmiotu będącego wnioskodawcą o rezygnacji z ubiegania się o finansowanie zadań badawczych w tym konkursie, albo
   b. zrezygnuję z przyjęcia finansowania zadań badawczych z innego źródła.
3. Oświadczam, że w przypadku przyznania decyzją Dyrektora NCN finansowania na zadania objęte wnioskiem :
   a. powiadomię osobę upoważnioną do reprezentowania podmiotu będącego wnioskodawcą o rezygnacji ze środków przyznanych na realizację zadań badawczych przez Dyrektora NCN w tym konkursie, albo
   b. zrezygnuję z ubiegania się o finansowanie zadań badawczych z innych źródeł.
4. Oświadczam, że jestem świadomy, że:
   a. podstawę prawną przetwarzania danych osobowych przez Narodowe Centrum Nauki stanowi art. 23 ust. 1 pkt 2 ustawy z dnia 29 sierpnia 1997 r. o ochronie danych osobowych (tekst jednolity: Dz. U. z 2002 r. Nr 101 poz. 926, ze zm.) – dane osobowe są niezbędne dla realizacji zadań określonych w ustawie z dnia 30 kwietnia 2010 r. o Narodowym Centrum Nauki;
   b. dane osobowe zawarte we wniosku o finansowanie projektu badawczego będą przetwarzane wyłącznie w celu dokonania jego ewaluacji, przeprowadzania ewaluacji realizacji zadań Centrum oraz upowszechniania informacji o ogłaszanych przez Centrum konkursach, a w przypadku przyznania środków finansowych na realizację projektu badawczego w celu nadzoru, kontroli, oceny realizacji i rozliczania projektu oraz sprawozdawczości;
   c. dane osobowe zawarte we wniosku zostaną udostępnione osobom, które na zlecenie Narodowego Centrum Nauki dokonują ewaluacji wniosku, lub uczestniczą w ewaluacji realizacji zadań Centrum, a w przypadku przyznania środków finansowych na realizację projektu także osobom, które uczestniczą w sprawowaniu nadzoru, kontroli oraz ocenie realizacji projektu badawczego i jego rozliczaniu;
   d. podanie danych jest dobrowolne, przy czym odmowa ich podania jest równoznaczna z nieprzekazaniem wniosku do oceny merytorycznej w konkursie;
   e. osoby, których dane są przetwarzane przez Narodowe Centrum Nauki mają prawo dostępu do treści swoich danych i ich poprawiania zgodnie z przepisami ustawy o ochronie danych osobowych.
5. Oświadczam, że projekt badawczy obejmuje badania:
   a. ~~wymagające zgody i / lub pozytywnej opinii właściwej komisji bioetycznej;~~
   b. ~~wymagające zgody właściwej komisji etycznej ds. doświadczeń na zwierzętach;~~
   c. ~~wymagające zgody na podstawie przepisów o organizmach genetycznie modyfikowanych;~~
   d. ~~wymagające zgody i / lub zezwolenia na badania na gatunkach chronionych lub na obszarach objętych ochroną;~~
   e. ~~wymagające innych pozwoleń, zgodnie z zasadami dobrej praktyki w danej dziedzinie / dyscyplinie naukowej;~~
   f. ~~kliniczne podlegające ustawie z dnia 6 września 2001 r. z późn. zm. o prawie farmaceutycznym lub ustawie z dnia 20 maja 2010 r. z późn. zm. o wyrobach medycznych.~~

   i zobowiązuję się do uzyskania wymaganych zgód, opinii, zezwoleń oraz pozwoleń przed rozpoczęciem realizacji badań, których dotyczą.

   Opis działań podjętych w celu zapewnienia wykonywania badań zgodnie z zasadami dobrej praktyki w danej dziedzinie / dyscyplinie naukowej oraz informacja czy takie zgody zostały już wydane, bądź informacje jak te warunki zostaną spełnione (maks. 2,5 tys. znaków ze spacjami).

bd.

6. Oświadczam, że jestem autorem / ~~współautorem~~ szczegółowego i skróconego opisu projektu badawczego w niniejszym wniosku.
7. Oświadczam, że:
   a. zapoznałem się z zasadami doręczania decyzji Dyrektora Narodowego Centrum Nauki;
   b. wyrażam zgodę na dokonanie weryfikacji wniosku przy pomocy oprogramowania antyplagiatowego oraz umieszczenie treści wniosku w bazie danych oprogramowania;
   c. zapoznałam/em się z treścią *Kodeksu Narodowego Centrum Nauki dotyczącego rzetelności badań naukowych i starania o fundusze na badania* i zobowiązuję się do jego stosowania.

G2. OŚWIADCZENIA OSOBY/OSÓB UPOWAŻNIONEJ/YCH DO REPREZENTOWANIA PODMIOTU BĘDĄCEGO WNIOSKODAWCĄ

Instytut Matematyczny Polskiej Akademii Nauk

1. Oświadczam, że zadania badawcze, objęte niniejszym wnioskiem, nie są i nie były finansowane z Narodowego Centrum Nauki, jak również z innego źródła. Oświadczam, że podmiot, który reprezentuję nie ubiega / ~~ubiega~~ się równocześnie o finansowanie zadań z innych źródeł.
2. Oświadczam, że w przypadku uzyskania finansowania na zadania objęte wnioskiem z innego źródła niezwłocznie poinformuję o tym fakcie Narodowe Centrum Nauki, i:
   a. zrezygnuję z ubiegania się o finansowanie zadań badawczych w tym konkursie, albo
   b. zrezygnuję z przyjęcia finansowania z innego źródła.
3. Oświadczam, że w przypadku przyznania decyzją Dyrektora NCN finansowania na zadania objęte wnioskiem:
   a. zrezygnuję ze środków przyznanych na realizację zadań badawczych przyznanych przez Dyrektora NCN w tym konkursie, albo
   b. zrezygnuję z ubiegania się o finansowanie zadań badawczych z innych źródeł.
4. Działając w imieniu podmiotu, który reprezentuję, w przypadku przyjęcia do finansowania zobowiązuję się do:
   a. włączenia projektu badawczego do planu zadaniowo-finansowego podmiotu;
   b. zatrudnienia kierownika projektu badawczego na zasadach zgodnych z wnioskiem i warunkami konkursu;
   c. zatrudniania wykonawców niezbędnych do realizacji projektu na zasadach zgodnych z wnioskiem i warunkami konkursu;
   d. zapewnienia warunków do realizacji prowadzonych badań, w tym udostępnienia przestrzeni biurowej/laboratoryjnej oraz aparatury naukowo badawczej niezbędnej do realizacji tych badań;
   e. zapewnienia obsługi administracyjno-finansowej realizacji projektu badawczego;
   f. sprawowania nadzoru nad realizacją projektu badawczego i prawidłowością wydatkowanych na ten cel środków finansowych.
5. Oświadczam, że projekt badawczy obejmuje badania:
   a. ~~wymagające zgody i / lub pozytywnej opinii właściwej komisji bioetycznej;~~
   b. ~~wymagające zgody właściwej komisji etycznej ds. doświadczeń na zwierzętach;~~
   c. ~~wymagające zgody na podstawie przepisów o organizmach genetycznie modyfikowanych;~~
   d. ~~wymagające zgody i / lub zezwolenia na badania na gatunkach chronionych lub na obszarach objętych ochroną;~~
   e. ~~wymagające innych pozwoleń, zgodnie z zasadami dobrej praktyki w danej dziedzinie / dyscyplinie naukowej;~~
   f. ~~kliniczne podlegające ustawie z dnia 6 września 2001 r. z późn. zm. o prawie farmaceutycznym lub ustawie z dnia 20 maja 2010 r. z późn. zm. o wyrobach medycznych.~~

   i zobowiązuję się do uzyskania wymaganych zgód, opinii, zezwoleń oraz pozwoleń przed rozpoczęciem realizacji badań, których dotyczą.

   Opis działań podjętych w celu zapewnienia wykonywania badań zgodnie z zasadami dobrej praktyki w danej dziedzinie / dyscyplinie naukowej oraz informacja czy takie zgody zostały już wydane, bądź informacje jak te warunki zostaną spełnione (maks. 2,5 tys. znaków ze spacjami).

bd.

6. W przypadku zakwalifikowania wniosku do finansowania wyrażam zgodę na zamieszczenie, wraz z informacją o wynikach konkursu, na stronie podmiotowej Narodowego Centrum Nauki oraz Ośrodka Przetwarzania Informacji (OPI) popularnonaukowego streszczenia projektu.
7. Działając w imieniu podmiotu, który reprezentuję oświadczam, że:
   a. zapoznałem się z zasadami doręczania decyzji Dyrektora Narodowego Centrum Nauki;
   b. zapoznałam/em się z treścią *Kodeksu Narodowego Centrum Nauki dotyczącego rzetelności badań naukowych i starania o fundusze na badania* i zobowiązuję się do jego stosowania;
   c. wyrażam zgodę na dokonanie weryfikacji wniosku przy pomocy oprogramowania antyplagiatowego oraz umieszczenie treści wniosku w bazie danych oprogramowania.

H1. SKRÓCONY OPIS PROJEKTU.

1. Research project objectives

1.1. **Limit shapes.** Many combinatorial structures can be viewed as *discrete versions of continuous geometric objects* such as 'surfaces' or 'shapes'. Good examples to keep in mind are provided by *Young diagrams*, *Young tableaux* (Figure 2a) [Sag01] as well as by *alternating sign matrices* [Pro01] (which appear in a somewhat concealed form on Figure 1a). It is then natural to ask about *the asymptotic behavior of these discrete shapes and surfaces as the size tends to infinity*. In many cases, the shape of the surface of the typical random element approaches a continuous limit. In such a case we say the model has a *"**limit shape**"*, see Figure 1b.

It should be stressed that such questions concerning the limit shape are usually formulated *in the probabilistic setup* in which the considered class of combinatorial objects is equipped with some natural probability measure.

1.2. **Dynamic limit shapes.** Many combinatorial structures can be naturally equipped with some sort of dynamics which gives rise to the questions about existence of some *"**dynamic limit shape**"* which is in our focus.

A good concrete example to keep in mind is the celebrated Schützenberger's *jeu de taquin*: from the initial standard Young tableau one removes the south-west corner box (which contains the number 1, see Figure 2a); in this way an empty cell (hole) is created. We either slide down the box directly above the hole or we slide left the box directly right to the hole, choosing the box which carries a smaller number. We continue the sliding; in this way the empty cell moves successively towards the boundary of the tableau, see Figure 2b.

Sample questions about the dynamical limit shape might be: *is there some typical trajectory along which the sliding occurs?* Suppose we iterate the above jeu de taquin transformation; *does this discrete model of draining a two-dimensional sandpile converge to some hydrodynamic limit?*

1.3. **The general research goal: dynamic asymptotic combinatorics.** The goal of this research proposal is to **investigate the asymptotics of large random combinatorial structures and their limit shapes** and to investigate their connection to (asymptotic) representation theory [Ker03], random matrix theory, Voiculescu's free probability theory [NS06], ergodic theory, statistical mechanics and mathematical physics.

Special attention will be given to **dynamical aspects** of such asymptotic behavior, in particular to **trajectories of particles** and **trajectories of second class particles** from the interacting particle systems [Lig05], **dynamical transformations** of large combinatorial structures and their **hydrodynamic limits**.

Specific examples which we have in mind are concentrated in (but not limited to) the class of combinatorial objects related to algebraic combinatorics and representation theory such as **random Young diagrams and tableaux** as well as combinatorial algorithms such as **Robinson–Schensted–Knuth algorithm** and **multidimensional Pitman transform** as well as **Schützenberger's jeu de taquin, promotion and evacuation**.

2. Significance of the project

2.1. **Limit shapes.** Questions concerning the asymptotic limit shape of random combinatorial objects are very natural and appealing to a wide range of mathematicians, as it can be seen by a variety of contexts in which they have been asked. In particular, **they were investigated by such mathematical celebrities as a Fields medal laureate Andrei Okounkov** [Oko06]**, Richard Stanley** [Sta07]**, Anatoly Vershik** [Ver95] **and the research in this area was appreciated by the mathematical community by invitations to give lectures on *International Congress of Mathematicians* by Philippe Biane** [Bia02] **and on *European Congress of Mathematics* by PI of this proposal** [Śni13]. There are several good reasons for studying such limit shapes:

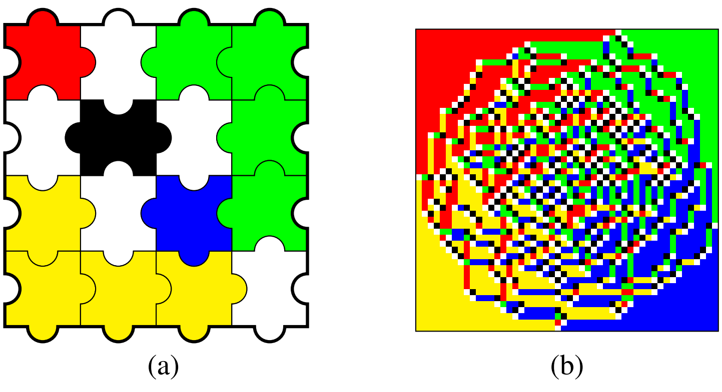
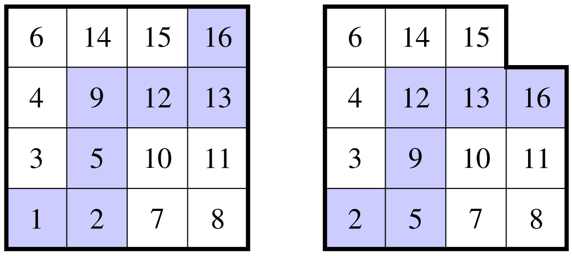


Figure 1. (a) A configuration of puzzles. The colors correspond to the six possible shapes of the tiles. (b) A larger configuration of puzzles chosen randomly, with the uniform distribution.



(a) (b)

Figure 2. (a) example of a standard Young tableau; the numbers are increasing along the rows and along the columns; (b) the outcome of slidings.

- Random combinatorial objects can be often considered as **models of mathematical physics** and, in particular, **statistical physics** [Ros81]; in this context it is indeed imperative to consider the large-scale limit and to investigate, for example, phase transitions.
- **Questions from group theory, harmonic analysis on groups, probability on groups or quantum information theory** (see, for example, [Dia03, MP02, SP12, MRŚ07]) often can be rephrased in the language of the character theory and thus **fit in the scope of the asymptotic representation theory**; if this is the case then understanding of the corresponding large random combinatorial structures is essential for answering the original question.
- There is also the **aesthetical motivation**: problems related to asymptotics of random combinatorial structures are often difficult and their solutions involve an appealing interface between seemingly distant disciplines of mathematics, such as combinatorics, analysis, harmonic analysis, ergodic theory, representation theory [BP99], probability theory, as well as quantum theory and statistical mechanics [RS01], see also the surveys [Oko06, Sta07, Ver95, Bia02, Śni13, Rom15].

There are several models for which a convergence towards some limit shape was proved [ST77, Ros81, Rom06, CK01, CEP96] or at least there are some heuristic arguments [CP10, Pro01, AHRV07].

2.2. **Random Young diagrams, random Young tableaux.** In this proposal we concentrate on examples with a representation-theoretic flavour; for this reason a special role is played by random Young diagrams and random Young tableaux. For the celebrated *Plancherel measure* the existence of the limit shape was proved by Logan and Shepp [LS77] as well as by Vershik and Kerov [VK77]. Kerov [Ker93] proved Central Limit Theorem for the fluctuations of the Young diagrams around the limit shape. These results were extended by Biane [Bia98, Bia01] and PI [Śni06a] for a very wide class of natural probability measures.

An analogue of the above result can be considered for the representations of the linear group $\mathrm{GL}_d(\mathbb{C})$. For various choices of the scaling the existence of the limit shape was proved by Biane, Bufetov, Gorin, Collins, Novak and the PI of the current research proposal [Bia95, CŚ09, BG15, CNŚ17].

2.3. **Dynamic problems.** One of the aspects which makes this research proposal pionieering is **the special emphasis on *dynamic problems in asymptotic combinatorics and their links with the ergodic theory*.** Such *dynamic problems* might involve large random combinatorial structures together with some natural trasformation or the dynamic aspects of some combinatorial algorithm. We shall discuss some natural examples below.

2.3.1. *RSK applied to a random input.* Robinson–Schensted–Knuth algorithm (RSK) is a fundamental object of algebraic combinatorics and representation theory of the symmetric group, in particular because of its link with *Littlewood–Richardson coefficients* [Sag01, FH91, Ful97]. Thus it is natural to ask the following quite general question: ***what can we say about various aspects of RSK when it is applied to a random input?*** Two concrete versions of this question were investigated by PI and Romik who proved *'asymptotic determinism of RSK insertion'* [RŚ15] as well as proved existence of limit shapes for the *bumping routes* [RŚ16].

2.3.2. *Jeu de taquin.* The main importance of the aforementioned Sützenberger's *jeu de taquin* [Sch77, Sag01, Ful97] is as a tool for studying the combinatorics of permutations and Young tableaux, especially with regards to the Robinson–Schensted–Knuth (RSK) algorithm and the representation theory of the symmetric groups $\mathfrak{S}_n$ and the linear groups $\mathrm{GL}_d(\mathbb{C})$.

Systematic investigation of dynamic limit shapes related to jeu de taquin (see Section 1.2) was initiated by PI and Romik [RŚ15, Śni14]. In the special case of the *Plancherel measure on the set of infinite Young tableaux* [RŚ15] we have fully described the dynamical system provided by jeu de taquin transformation from the viewpoint of the ergodic theory. The key component in the proof was to investigate the **limit shape of the trajectory traversed by the empty cell** (Figure 2), called *jeu de taquin trajectory*.

2.3.3. *Mathematical physics: second class particles.* Diffusive systems are often described by hydrodynamical equations (which are nonlinear PDEs), which can develop *shocks*, i.e. singularities in the solution. In order to describe the dynamics of shocks on the microscopic level it is convenient to use ***second class particles*** [Lig05].

Ferrari and Kipnis [FK95] as well as Mountford and Guiol [MG05] proved that **the speed of a second class particle in TASEP (*Totally AntySymmetric Exclusion Process*)** [Spi70] **almost surely converges to a finite random limit**. Romik and PI [RŚ15] proved an analogous result for a version of TASEP in which the jump probabilities are given by a representation-theoretic *Plancherel growth process*.

2.4. **Random walks and their combinatorics.** Dykema and Haagerup [DH04] stated as a conjecture an infinite number of combinatorial identities which generalize the classical result of Cauchy

$$2^{2k} = \sum_{p+q=k} \binom{2p}{p}\binom{2q}{q}. \tag{1}$$

This classical identity and one of its bijective proofs have a natural interpretation in the language of random walks on the one-dimensional lattice $\mathbb{Z}$ and their limit object. More specifically, the left-hand side of (1) (in the limit as $k \to \infty$ tends to infinity) is related to the *Brownian motion*. The first factor on the right-hand side is related to the *Brownian bridge* [IM74] while the second factor is related to the *Brownian excursion* [IM74]. The bijection turns out to behave in a continuous way with respect to the limit procedure and converges to the transformation introduced by Pitman [Pit75] in a different context. The bijection and its limit essentially establish a measure-preserving transformation between a Brownian bridge and a Brownian excursion.

This simple bijection can be described using a fancy language as application of Robinson–Schensted–Knuth algorithm to a word in an alphabet which consists of just two letters. By removal of the restriction on the size of the alphabet Biane, Bougerol and O'Connell [BBO05] generalized the aforementioned result to random walks on higher-dimensional lattices within the Weyl chamber, non-colliding random walks and the corresponding Brownian motions, as well as higher-dimensional analogues of Pitman transform.

3. Work plan

3.1. **Research goal (A): RSK applied to a random input.** Consider RSK algorithm applied to an i.i.d. sequence of letters. One of the goals of this proposal is to answer the asymptotic version of the following question: ***what can we say about the time evolution of the insertion tableau*** *as we are inserting more and more letters?* The letters are inserted at the bottom of the tableau and then they are bumped up and to the left by other letters; this results with a random, time-dependent transformation of (a finite part) of the lattice $\mathbb{N}^2$ corresponding to the positions of the boxes. **We plan to show that with the right choice of the scaling this 'semigroup' of random transformations converges to a specific deterministic semigroup of transformation of the quarterplane $\mathbb{R}_+^2$ which can be interpreted as dynamics of an incompressible fluid.**

3.2. **Research goal (B): jeu de taquin.** Consider a random standard Young tableau filling a large Young diagram with some macroscopic limit shape (say, a rectangle). **We plan to show that asymptotically the trajectory traversed by the empty box in Schützenberger's algorithm (the jeu de taquin trajectory) converges to a random curve.** This result would be an analogue of the aforementioned result of Romik and PI [RŚ15] for a much wider class of probability measures on the set of Young tableaux.

Consider a random standard Young tableau just as above and let us apply the jeu de taquin transformation several times. This can be viewed as a *discrete version of draining a two-dimensional container with sand grains*. **We plan to prove that such random sandpile dynamics on the subset of the lattice $\mathbb{N}^2$ converges to a deterministic semigroup on a subset of $\mathbb{R}_+^2$.**

3.3. **Research goal (C): TASEP with the uniform history distribution.** Consider a **TASEP system** with a finite numer of particles and **with a specified initial and the final state**. The time evolution of such a system (its *history*) consists of a finite number of steps in which a prescribed particle makes a jump. We consider a version of TASEP dynamics with modified jump probabilities in which **to each history we associate equal probability**. One of the specific goals in this proposal is to **prove a dynamic limit shape theorem for the trajectories of second class particles**.

3.4. **Research goal (D). Brownian motion limit of Dykema–Haagerup identities.** The structure of the left-hand side of more general Dykema–Haagerup identities than (1) suggests that they also have a very natural combinatorial interpretation in terms of random walks on higher-dimensional lattices. However, in this case the right-hand side counts more complicated combinatorial objects which as limits have more sophisticated multidimensional versions of Brownian bridges and excursions. The bijective proof of Dykema–Haagerup identities which was found by PI [Śni06b] also seems to involve a version of Robinson–Schensted–Knuth algorithm but a much more complex one in which the given word is parsed several times and after each iteration the natural direction of time is reversed. One of the specific goals of the project is **to check if this bijection has a continuous measure-preserving version (á la Pitman) and to investigate what kind of information about the mutlidimensional Brownian motion is provided by this new map** and **to explore its applications to multidimensional Brownian motions, Brownian bridges, Brownian excursions, non-colliding Brownian motions**.

3.5. **Research goal (E). Other problems.** This research project has a quite wide scope of investigating (dynamic) limit shapes of various large random combinatorial structures, for this reason we also plan to attack some other open problems, not discussed in datail in Section 2.

One of specific examples which we have in mind is proving the conjectures concerning the dynamical limit shapes for particles in uniformly random sorting networks [AHRV07]. This part of the research proposal is quite risky and we are not sure if this approach will turn out succesful.

We also plan to explore further applications of the asymptotic analysis of the characters of the linear groups $\mathrm{GL}_d(\mathbb{C})$ [GP15].

4. Research methodology

Our main tools will be **the combinatorial methods and objects related to *the asymptotic representation theory of the symmetric groups***, in particular *the Kerov–Olshanski algebra of polynomial functions on the set of Young diagrams* [KO94], *Jucys–Murphy elements* [CSST14], *Kerov character polynomials* [DFŚ10, Śni13]. We also plan to use some results and ideas from the previous work of PI with Romik [RŚ15].

We will also use the general methods of **the analytic combinatorics**, **the algebraic combinatorics** and **the enumerative combinatorics** as well as **the combinatorial methods of the random matrix theory** [NS06].

The trajectories of jeu de taquin seem to be examples of anamalous diffusions with rather elusive probabilistic characteristics. For this reason in the initial phase of the project we plan to run a series of computer simulations on large Young diagrams in order to get a heuristic insight, for example on the question about the (non)gaussianity of the shape fluctuations.

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H2. SZCZEGÓŁOWY OPIS PROJEKTU.

DYNAMIC ASYMPTOTIC COMBINATORICS

PIOTR ŚNIADY

1. RESEARCH PROJECT OBJECTIVES

1.1. **Combinatorics vs asymptotic combinatorics.** The main subject of *combinatorics* is the investigation of finite discrete structures. A good concrete example to keep in mind are *integer partitions* of a given integer n, such as $5 = 4 + 1$, see Figure 1a. A more sophisticated example of such a finite discrete structure is a tiling of a square area $n \times n$ (having a specific configuration of the knobs along the boundary) with square tiles, each having exactly two convex knobs and two concave knobs, see Figure 1b. Typical questions of combinatorics involve enumeration ('how many tilings there are?') and bijections which would explain coincidences in cardinalities of various classes of objects.

The goal of *asymptotic combinatorics* — which happens also to be the goal of this proposal — is to investigate the combinatorial structures and algorithms *in the limit as their size tends to infinity* [Ver95]. This change of perspective has

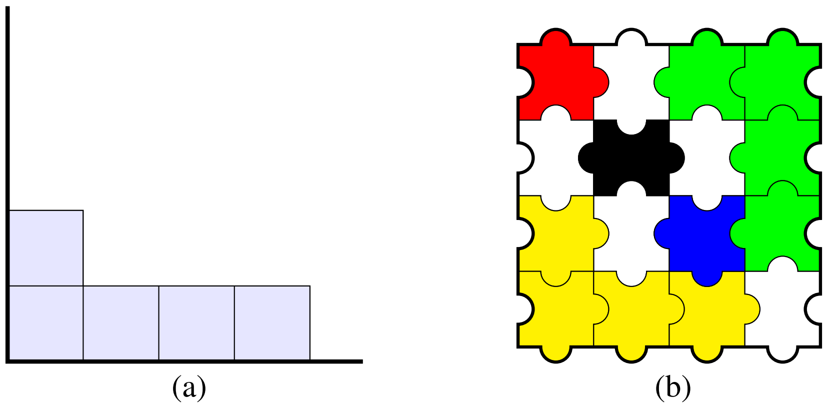


Figure 1. (a) Young diagram, a graphical representation of a partition $5 = 4 + 1$. (b) A configuration of puzzles. The colors correspond to the six possible shapes of the tiles.

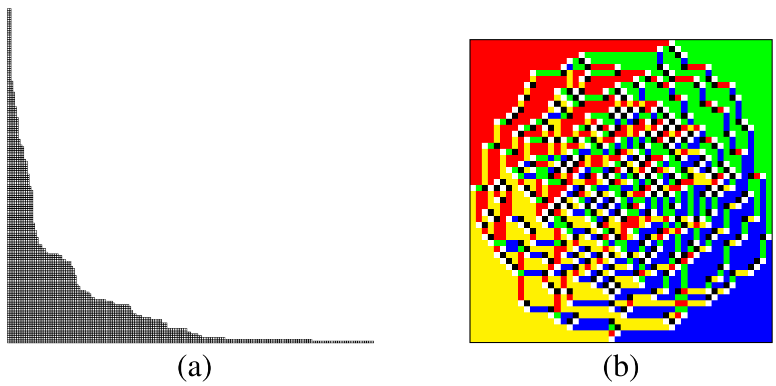


Figure 2. Larger versions of the combinatorial objects from Figure 1. The shown objects were chosen randomly, with the uniform distribution. The individual boxes were not shown. For many models (such as the ones above) there exists a *limit shape* for large random objects.

profound implications on the type of questions which one considers; we shall discuss it below on the specific example of *the representation theory*.

1.2. **The key example: asymptotic representation theory.** For almost all questions of the representation theory of the symmetric groups $\mathfrak{S}_n$ there is a well-known answer given by some combinatorial objects such as the above-mentioned integer partitions (also known as *Young diagrams*, see Figure 1a) and *Young tableaux* (Figure 3a) and some combinatorial algorithms such as *Littlewood–Richardson rule* [Sag01] and this level of understanding is fully satisfactory from the point of view of the classical combinatorics. Regretfully, as $n \to \infty$ and the size of the group tends to infinity, such combinatorial answers become quickly cumbersome and too complicated to give a tractable answer [Bia98].

In the case of the Littlewood–Richardson rule (which gives the multiplicities of a decomposition of a certain natural reducible representation of the symmetric group $\mathfrak{S}_n$ into the irreducible components) the asymptotic combinatorics would call for investigation of the **probabilistic version of the original combinatorial problem**. More specifically, this means that one should *consider some natural probability measure on the set of the combinatorial objects in question* (in this case: the set of irreducible representations or the set of Young diagrams) and *investigate the statistical properties of the corresponding* ***random combinatorial object***, for example from the viewpoint of some law of large numbers.

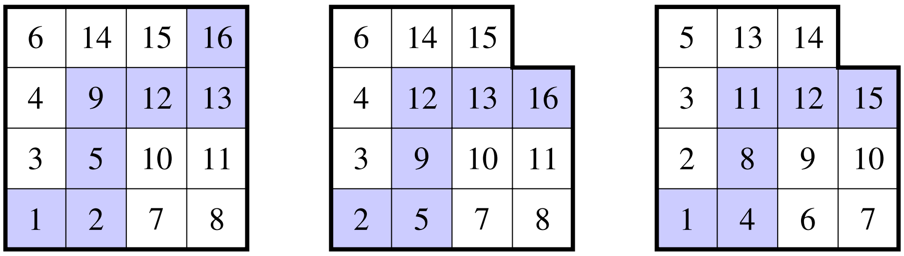
1.3. **Limit shapes.** Many combinatorial structures can be viewed as *discrete versions of continuous geometric objects* such as 'surfaces' or 'shapes', therefore they can be embedded into this or another space of such geometric objects. Good examples to keep in mind are provided by the above-mentioned *Young diagrams* (Figure 1a) and *Young tableaux* (Figure 3a) [Sag01] as well as by *alternating sign matrices* [Pro01] (which appear in a somewhat concealed form on Figure 1b). It is then natural to ask about *the asymptotic behavior of these discrete shapes and surfaces as the size parameter tends to infinity*. In many cases, the shape of the surface of the typical random element approaches a continuous limit. In such a case we say the model has a *"**limit shape**"*. This type of phenomenon is illustrated by computer simulations on Figure 2.

It should be stressed that such questions concerning the limit shape are usually formulated *in the probabilistic setup* (which we discussed briefly in Section 1.2 above) in which the considered class of combinatorial objects is equipped with some natural probability measure. In particular, the above-mentioned *'convergence to a limit shape'* should be understood in some sense provided by the probability theory (for example, like in Weak Law of Large Numbers, *'in probability'*).

1.4. **Dynamics.** Many combinatorial structures can be equipped with some sort of dynamics. One of the ways to do it is to introduce some transformation of a given class of structures.

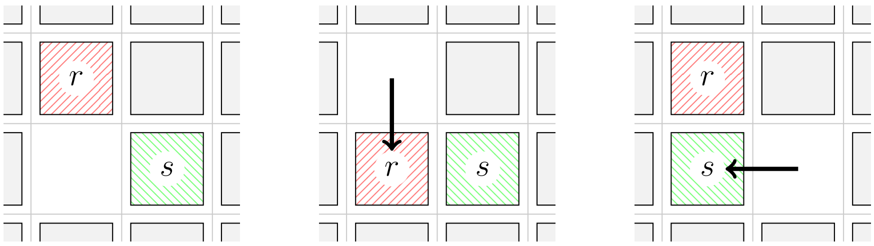
A good concrete example to keep in mind is the celebrated Schützenberger's *jeu de taquin* [Sch77, Sag01, Ful97]. It is a transformation on the set of *skew (semi)standard Young tableaux*. Its main importance is as a tool for studying the combinatorics of permutations and Young tableaux, especially with regards to the Robinson–Schensted–Knuth (RSK) algorithm and the representation theory of the symmetric groups $\mathfrak{S}_n$ and the linear groups $\mathrm{GL}_d(\mathbb{C})$.

Jeu de taquin can be adapted to the framework of *standard Young tableaux*, as follows. From the initial standard tableau one removes the south-west corner box (which necessarily contains the number 1, see Figure 3a); in this way an empty cell is created. We begin sliding of the boxes according to the rules depicted on Figure 4.



(a) (b) (c)

Figure 3. (a) example of a standard Young tableau; the numbers are increasing along the rows and along the columns; (b) the outcome of slidings; (c) in the final step we subtract 1 from each box. The highlighted boxes form the path along which the sliding occurred.



(a) (b) (c)

Figure 4. Elementary step of the jeu de taquin transformation: (a) the initial configuration of boxes, (b) the outcome of the slide in the case when $r < s$, (c) the outcome of the slide in the case when $s < r$.

In this way the empty cell moves successively towards the boundary of the tableau, see Figure 3b. In the final step we subtract 1 from each entry, see Figure 3c; in this way a standard Young tableau is created. In this way jeu de taquin is a transformation on the set of standard Young tableaux; alternatively one can define jeu de taquin as a transformation on the set of *infinite* standard Young tableaux.

1.5. **Dynamic limit shapes.** Investigation of various kinds of dynamics on large random combinatorial structures naturally gives rise to the questions about existence of some *"**dynamic limit shape**"* which is in the focus of this proposal.

More specifically, in our guiding example of Schützenberger's jeu de taquin, one could ask the following concrete questions about such dynamic limit shapes.

(Q1) For a randomly chosen Young tableau filling some prescribed large Young diagram, *does the jeu de taquin trajectory have some limit shape*? This kind of question is illustrated by a computer simulation on Figure 5.

(Q2) Suppose we pursue the dynamical viewpoint even further by iterating our transformation and consider a time evolution of our discrete combinatorial structure. In the case of jeu de taquin this would correspond to a discrete model of draining a two-dimensional liquid from a container: the particles diffuse to the bottom and to the left and then they disappaer in the corner. *Is there some deterministic hydrodynamic limit of this time evolution of the boxes?*

1.6. **The general research goal: dynamic asymptotic combinatorics.** The goal of this research proposal is to **investigate the asymptotics of large random combinatorial structures and their limit shapes** and **to investigate their connection to (asymptotic) representation theory** [Ker03]**, random matrix theory, free probability**

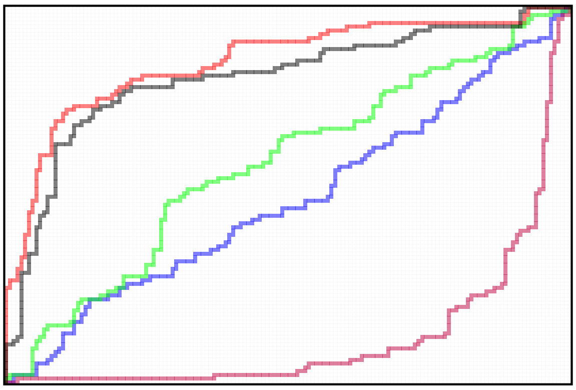


Figure 5. Sample jeu de taquin trajectories for a large rectangle.

theory (in the sense of Voiculescu) [NS06]**, ergodic theory, geometric complexity theory** [Lan16]**, statistical mechanics and mathematical physics**. We plan to prove probabilistic results such as Law of Large Numbers and Central Limit Theorem which govern behavior of such large random combinatorial objects and algorithms.

Special attention will be given to **dynamical aspects** of such asymptotic behavior, in particular to asymptotics of **trajectories of particles** and **trajectories of second class particles** from the interacting particle systems [Lig05], **dynamical transformations** (in the flavor of the ergodic theory) of large combinatorial structures and their **hydrodynamic limits**.

Specific examples which we have in mind are concentrated in (but not limited to) the class of combinatorial objects related to algebraic combinatorics and representation theory such as **random Young diagrams, random Young tableaux** as well as in the analogous class of combinatorial algorithms such as **Robinson–Schensted–Knuth algorithm** and **multidimensional Pitman transform (also with time swapping)** as well as **Schützenberger's jeu de taquin, promotion and evacuation**.

We will highlight some selected specific topics which will be covered by the proposal below in Sections 2.3.2, 2.3.4, 2.5, 2.6 below after we present the context.

2. Significance of the project

2.1. **Limit shapes.** Questions concerning the asymptotic limit shape of random combinatorial objects (which we already discussed in Section 1.3) are very natural and appealing to a wide range of mathematicians, as it can be seen by a variety of contexts in which they have been asked. In particular, **they were investigated by such mathematical celebrities as a Fields medal laureate Andrei Okounkov** [Oko06]**, Richard Stanley** [Sta07]**, Anatoly Vershik** [Ver95] **and the research in this area was appreciated by the mathematical community by invitations to give lectures on *International Congress of Mathematicians* by Philippe Biane** [Bia02] **and on *European Congress of Mathematics* by PI of this proposal** [Śni13]. In the following we shall discuss briefly a few selected highlights from this field.

The list of models for which a convergence towards some limit shape is known to hold true includes:

- random partitions, sampled with the uniform distribution [ST77] (Figure 2a),
- random partitions, sampled according to *the corner growth model* [Ros81],
- random *extremal Erdős–Szekeres permutations* [Rom06],
- uniformly random *plane partitions* [CK01],
- random *domino tilings of the Aztec diamond* [CEP96].

There are also interesting examples for which proving the convergence remains an open, challenging problem:

- the random *alternating sign matrices* (Figures 1b and 2b) [CP10, Pro01],
- random *sorting networks* [AHRV07].

There are several good reasons for studying such limit shapes; we overview them briefly in the following.

- Random combinatorial objects can be often considered as **models of mathematical physics** and, in particular, **statistical physics**; in this context it is indeed imperative to consider the large-scale limit and to investigate, for example, phase transitions.
- **Questions from group theory, harmonic analysis on groups, probability on groups or quantum information theory** (see, for example, [Dia03, MP02, SP12, MRŚ07]) often can be rephrased in the language of the character theory and thus **fit in the scope of the asymptotic representation theory**; if this is the case then understanding of the corresponding large random combinatorial structures is essential for answering the original question.
- There is also **the aesthetical motivation** which is twofold. Firstly, computer simulations often give rise to beautiful pictures analogous to Figure 2b. Secondly, and more importantly, problems related to asymptotics of random combinatorial structures are often difficult and their solutions involve an appealing interface between seemingly distant disciplines of mathematics, such as combinatorics, analysis, harmonic analysis, ergodic theory, representation theory [BP99], probability theory, as well as quantum theory and statistical mechanics [RS01], see also the surveys [Oko06, Sta07, Ver95, Bia02, Śni13, Rom15].

We shall discuss some additional motivations later in Section 2.2.3.

2.2. **Random Young diagrams and tableaux.** In this research proposal we put additional attention to the viewpoint of the representation theory. For this reason a special role is played by two special classes of combinatorial objects: *Young diagrams* (Figure 1a) and *Young tableaux* (Figure 3a) which are intimately related both to the representation theory of the symmetric groups $\mathfrak{S}_n$ and the representation theory of the linear groups GL_d [FH91].

2.2.1. *Probability measure associated to a representation.* With this viewpoint in mind it is beneficial to replace the uniform probability measure (which appears in most of the examples from Section 2.1) by another probability distribution which is tailored specifically for a given problem. This construction is an example of *'replacing a difficult combinatorial problem by its simpler probabilistic version'* which we mentioned in Section 1.2. The starting point is some given reducible representation V of some group G with the decomposition into irreducible components given by

$$V = \bigoplus_{\zeta \in \widehat{G}} n_\zeta V^\zeta, \tag{1}$$

where $n_\zeta \in \{0, 1, \dots\}$ denotes the multiplicity of a given irreducible component V^ζ inside V. A typical problem of classical combinatorics would be to 'understand' the coefficients n_ζ which — as we discussed in Section 1.2 on the example of the Murnaghan–Nakayama rule — might be computationally intractable. An analogous problem of asymptotic combinatorics is more modest: to 'understand' the basic statistical properties of a *random irreducible representation* ζ, sampled with the distribution

$$\mathbb{P}(\zeta) := \frac{n_\zeta \dim V^\zeta}{\dim V} \qquad \text{for } \zeta \in \widehat{G}. \tag{2}$$

In the examples which we consider, such an irreducible representation can be identified with a Young diagram, thus the latter problem fits perfectly into the general framework which we consider in this proposal, namely *random combinatorial objects and their limit shapes*.

2.2.2. *Examples of reducible representations and the corresponding Young diagrams.* There are several interesting examples of reducible representations for which existence of the limit shapes for the corresponding Young diagrams was proved.

- For the left-regular representation $\ell^2(\mathfrak{S}_n)$ of the symmetric group $\mathfrak{S}_n$, the corresponding measure $\mathbb{P}$ from (2) is the celebrated *Plancherel measure*. The existence of the limit shape for the corresponding random Young diagrams for $n \to \infty$ was proved by Logan and Shepp [LS77] as well as by Vershik and Kerov [VK77]. Later on Kerov [Ker93] proved Central Limit Theorem for the fluctuations of the Young diagrams around the limit shape.
- Biane [Bia98, Bia01] proved existence of the limit shapes for a very large class of representations of the symmetric groups and he proved that this class is closed under natural representation-theoretic operations such as induction, restriction, outer product and tensor product. Biane also found a deep connection between the form of the limit shapes and Voiculescu's free probability theory [NS06]. The PI of the current proposal [Śni06a] improved Biane's Law of Large Numbers as well as Kerov's CLT by showing that the fluctuations of the Young diagrams from this wide class around the limit shape are Gaussian.
- An analogue of the above result of Biane can be considered for the representations of the linear group $\mathrm{GL}_d(\mathbb{C})$: suppose that two irreducible representations which correspond to large Young diagrams are given; what can we say about the limit shape of Young diagrams which correspond to the tensor product of the initial representations? This kind of result is more subtle as it depends on the choice of scaling at which the Young diagrams diverge to infinity as the rank the rank of the group $d \to \infty$ tends to infinity; for various choices of the scaling the existence of the limit shape was proved by Biane, Bufetov, Gorin, Collins, Novak and the PI of the current research proposal [Bia95, CŚ09, BG15, CNŚ17].

2.2.3. *The Young graph and its boundary.* Another motivation for studying asymptotics of random Young diagrams and Young tableaux comes from **the harmonic analysis on *the infinite symmetric group*** $\mathfrak{S}_\infty$. More specifically, Vershik and Kerov [VK81] found a new conceptual proof of Thoma's classification of the extremal characters of $\mathfrak{S}_\infty$ [Tho64] which was based on an observation that there is a bijection — which is a refined version of the construction from Section 2.2.1 — between the set of the characters of the infinite symmetric group $\mathfrak{S}_\infty$ and a certain class of probability measures on the set of infinite paths in the *Young graph* (cf. Figure 6) or, equivalently, on the set of an *infinite standard Young tableaux*. In this way a link between an interesting class of random (infinite) Young tableaux, the *boundary of the Young graph* (in the sense of Martin or the minimal boundary) and the harmonic analysis on the infinite symmetric group $\mathfrak{S}_\infty$ was established.

2.3. **Dynamic problems.** One of the aspects which makes this research proposal pionieering is **the special emphasis on *dynamic problems in asymptotic combinatorics and their links with the ergodic theory*.** Such *dynamic problems* might involve large random combinatorial structures together with some natural trasformation or the dynamic aspects of some combinatorial algorithm. We shall discuss some natural examples below.

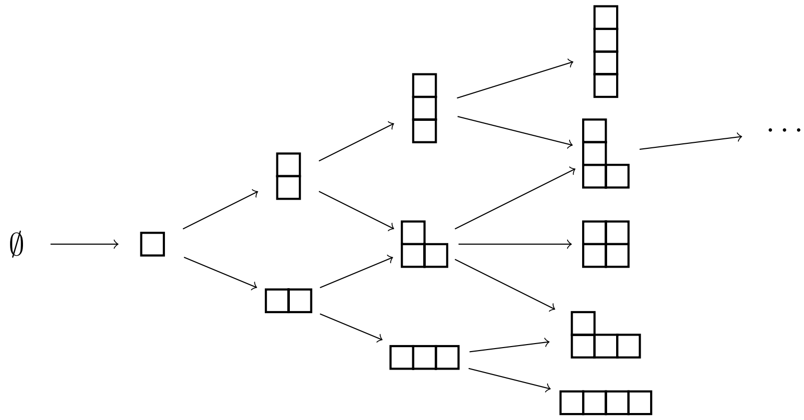


Figure 6. The Young graph. Its vertices are Young diagrams; edges connect pairs of diagrams which differ by addition or deletion of a single box.

2.3.1. *Robinson–Schensted–Knuth algorithm applied to a random input.* Robinson–Schensted–Knuth algorithm (RSK) is a fundamental object of algebraic combinatorics and representation theory of the symmetric group, in particular because of its link with *Littlewood–Richardson coefficients* [FH91].

This algorithm is a bijection which takes as an input a *word* (=a finite sequence of *letters*, i.e. elements of an *alphabet*=some linearly ordered set) and produces as an output a pair of tableaux: the *insertion tableau* and the *recording tableau*.

The insertion tableau is a semistandard tableau constructed as follows: we start with the empty tableau and perform a number of *insertions* in which consecutive letters are being inserted into the insertion tableau. Each insertion is computed by performing a succession of *bumping steps* whereby the letter is inserted into the first row of the tableau (as far to the right as possible so that the row remains increasing and no gaps are created), bumping an existing entry from the first row into the second row, which results in an entry of the second row being bumped to the third row, and so on, until finally the entry being bumped settles down in an unoccupied position. An example is shown in Figure 7.

The *recording tableau* is a standard tableau which contains the information about the order in which new entries appeared in the insertion tableau.

RSK has also an infinite version in which the input is an *infinite* sequence and the output is only the recording tableau (which now becomes an *infinite Young tableau*).

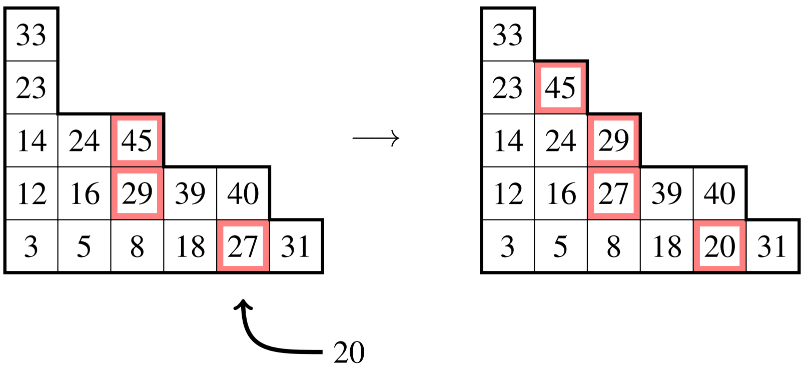


Figure 7. Elementary step of RSK algorithm: a new entry is inserted into the *insertion tableau*. The highlighted entries form the *bumping route*.

From the viewpoint of algebraic combinatorics it is natural to ask the following quite general question: ***what can we say about various aspects of RSK when it is applied to a random input?*** A significant part of this proposal is devoted to answering various versions of this question. Before presenting the details of our research goal we shall summarize below some known results.

Kerov and Vershik [VK86] investigated the recording tableau associated to an infinite sequence of independent, identically distributed (i.i.d.) random letters selected from some alphabet. They proved that the distribution of such an infinite tableau coincides with the probability distribution associated to a specific Thoma's extremal character of the infinite symmetric group $\mathfrak{S}_\infty$ — the latter characters were already discussed in Section 2.2.3. A special important example is the one in which the probability distribution on the alphabet has no atoms; the resulting probability measure on the set of infinite Young tableaux is the celebrated *Plancherel measure*. It follows that the aforementioned results on the limit shape of random Young diagrams [LS77, VK77] also apply to the outcome of RSK applied to such a random input.

Romik and PI of this proposal [RŚ16] investigated the above-mentioned special case when RSK is applied to an i.i.d. sequence of letters with a non-atomic distribution; we proved that the *bumping routes* (cf. Figure 7) converge to a limit shape.

2.3.2. *Specific research goal (A): RSK applied to a random input.* Consider RSK algorithm applied to an i.i.d. sequence of letters selected from some alphabet without atoms. One of the goals of this proposal is to answer the asymptotic version of the following question: ***what can we say about the time evolution of the insertion tableau*** *as we are inserting more and more letters from the word?* Clearly, the letters are inserted at the bottom of the tableau and then they are bumped up and to the left by other letters; this results with a random, time-dependent transformation of (a finite part) of the lattice $\mathbb{N}^2$ corresponding to the positions of the boxes. **We plan to show that with the right choice of the scaling this 'semigroup' of random transformations converges to a specific deterministic semigroup of transformation of the quarterplane $\mathbb{R}_+^2$ which can be interpreted as dynamics of an incompressible fluid.**

In a rare moment of sincerity we must admit that we do not see any deep applications of this result to the outer world and our only motivation is that this is a really cool and natural problem.

2.3.3. *Jeu de taquin.* We continue discussion of Schützenberger's jeu de taquin (see Section 1.4). Systematic investigation of dynamic limit shapes related to jeu de taquin (see Section 1.5) was initiated by PI and Romik [RŚ15, Śni14]. In the special case of the *Plancherel measure on the set of infinite Young tableaux* [RŚ15] we have fully described the dynamical system provided by jeu de taquin transformation from the viewpoint of the ergodic theory. The key component in the proof was to investigate the **limit shape of the trajectory traversed by the empty cell** (indicated on Figure 3 by the highlighted boxes), called *jeu de taquin trajectory*.

2.3.4. *Specific research goal (B): jeu de taquin.* Consider a random standard Young tableau filling a large Young diagram with some macroscopic limit shape (say, a rectangle). **We plan to show that asymptotically the trajectory traversed by the empty box in Schützenberger's algorithm (the jeu de taquin trajectory) converges to a random curve.** This type of phenomenon is illustracted by a computer simulation on Figure 5. This result would be an analogue of the aforementioned result of Romik and PI [RŚ15] for a much wider class of probability measures on the set of Young tableaux.

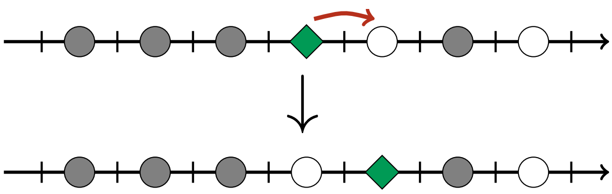
Consider a random standard Young tableau filling a large Young diagram with some macroscopic limit shape (say, a rectangle). Let us apply the jeu de taquin transformation several times. This can be viewed as a *discrete version of draining a two-dimensional container with sand grains*, see question (Q2) from Section 1.5. **We plan to prove** an analogue of a result claimed in Section 2.3.2 above **that such random sandpile dynamics on the subset of the lattice $\mathbb{N}^2$ converges to a deterministic semigroup on a subset of $\mathbb{R}_+^2$.**

2.4. **Mathematical physics: second class particles.** In this research proposal we are concerned with variations of TASEP (*Totally AntySymmetric Exclusion Process*) which is the default stochastic model for transport phenomena [Spi70]. It is an interacting particle particle system on the integer lattice $\mathbb{Z}$ in which each node can be either empty or occupied by a single particle. At each time step a single particle can jump one node to the right, provided that this adjacent node is empty. The particle which jumps is selected randomly and the choice of the jump probabilities depends on the details of a model.

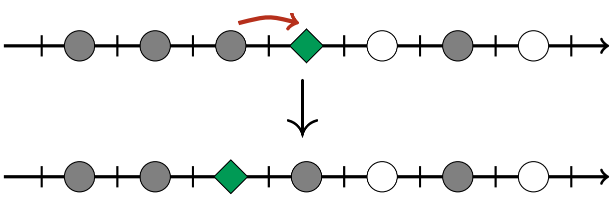
Diffusive systems (including TASEP) are often described by hydrodynamical equations (which are nonlinear partial differential equations), which can develop *shocks*, i.e. singularities in the solution. Whereas the large scale continuous description of shocks is well established, much less is known about the microscopic description [BFKR10]. The first difficulty one has to overcome is to define the position of the shock on the lattice lengthscale; one possible solution is to use the concept of ***second class particles*** which we present below.

Let us declare that one of the particles is a ***second class particle***. Just like a *second class passenger* who has to yield a seat to any *first class passenger*, such a second class particle has to jump to the left if a particle from the left attempts to jump to the right, see Figure 8b. An alternative viewpoint is declare that a second class particle is *a pair which consists of an particle on the left and an empty node on the right*, see Figure 9. Second class particles follow the trajectories of density fluctuations, are attracted by shocks and therefore serve as good markers for the shock position.

Ferrari and Kipnis [FK95] as well as Mountford and Guiol [MG05] proved that **the speed of a second class particle in TASEP almost surely converges to a finite random limit**.



(a)



(b)

Figure 8. A particle system with a second class particle (represented as a diamond) and the allowed transitions involving the second class particle. Other transitions obey the TASEP rules.

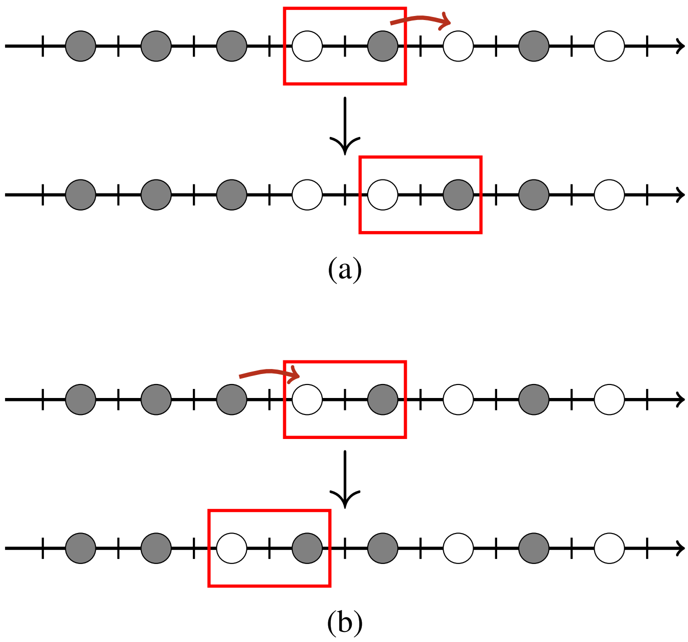


Figure 9. A second class particle viewed as a pair consisting of a particle and an empty node and its transitions.

An analogous result holds true for a version of TASEP in which the jump probabilities are given by a representation-theoretic *Plancherel growth process*; the latter result turns out to be equivalent to the aforementioned result of Romik and PI of this proposal [RŚ15] about the limit shape of jeu de taquin trajectories (see Section 2.3.3).

2.5. **Specific research goal (C): TASEP with the uniform history distribution.** Consider a **TASEP system** with a finite numer of particles and **with a specified initial and the final state**. The time evolution of such a system (its *history*) consists of a finite number of steps in which a prescribed particle makes a jump. We consider a version of TASEP dynamics with modified jump probabilities in which **to each history we associate equal probability**. One of the specific goals in this proposal is to **prove a dynamic limit shape theorem for the trajectories of second class particles**.

This model has an interesting feature: by modifying the boundary conditions it is possble to obtain several *rarefaction fans* [FK95] which is a feature which might be interesting from the viewpoint of mathematical physics.

2.6. **Specific research goal (D): Brownian motion limit of Dykema–Haagerup identities.** Dykema and Haagerup [DH04] stated as a conjecture an infinite number of combinatorial identities which generalize the classical result of Cauchy

$$2^{2k} = \sum_{p+q=k} \binom{2p}{p} \binom{2q}{q}. \tag{3}$$

The simplest of these new *Dykema–Haagerup identities* is

$$3^{3k} = \sum_{p+q=k} \binom{3p}{p,p,p} \binom{3q}{q,q,q} + 3 \sum_{\substack{p+q+r=k-1 \\ r'+q'=r+q+1 \\ p''+r''=p+r+1}} \binom{2p+p''}{p,p,p''} \binom{2q+q'}{q,q,q'} \binom{r+r'+r''}{r,r',r''}. \tag{4}$$

The classical identity (3) and one of its bijective proofs have a natural interpretation in the language of random walks on the one-dimensional lattice $\mathbb{Z}$ and their limit object. More specifically, the left-hand side of (3) (in the limit as $k \to \infty$ tends to infinity) is related to the *Brownian motion*. The first factor on the right-hand side is related to the *Brownian bridge* [IM74] while the second factor is related to the *Brownian excursion* [IM74]. The bijection turns out to behave in a continuous way with respect to the limit procedure and converges to the transformation introduced by Pitman [Pit75] in a different context. The bijection and its limit essentially establish a measure-preserving transformation between a Brownian bridge and a Brownian excursion.

This simple bijection can be described using a fancy language as application of Robinson–Schensted–Knuth algorithm to a word in an alphabet which consists of just two letters. By removal of the restriction on the size of the alphabet Biane, Bougerol and O'Connell [BBO05] generalized the aforementioned result to random walks on higher-dimensional lattices within the Weyl chamber, non-colliding random walks and the corresponding Brownian motions, as well as higher-dimensional analogues of Pitman transform.

The structure of the left-hand side of Dykema–Haagerup identities (such as (4)) suggests that they also have a very natural combinatorial interpretation in terms of random walks on higher-dimensional lattices. However, in this case the right-hand side counts more complicated combinatorial objects which as limits have more sophisticated multidimensional versions of Brownian bridges and excursions. The bijective proof of Dykema–Haagerup identities which was found by PI [Śni06b] also seems to involve a version of Robinson–Schensted–Knuth algorithm but a much more complex one in which the given word is parsed several times and after each iteration the natural direction of time is reversed. One of the specific goals of the project is **to check if this bijection has a continuous measure-preserving version (á la Pitman) and to investigate what kind of information about the mutlidimensional Brownian motion is provided by this new map** and **to explore its applications to multidimensional Brownian motions, Brownian bridges, Brownian excursions, non-colliding Brownian motions**.

3. Work plan

3.1. **Research goal (A): RSK applied to a random input.** We continue discussion from Section 2.3.2.

One of the technical results contained in the aforementioned work of Romik and PI [RŚ15] was *asymptotic determinism of RSK insertion* and we think that the full power of this result has not been exploited yet. In [RŚ16] we explained how to apply this result in another context to investigate the limit shape of jeu de taquin trajectories. It seems that by applying some symmetries of RSK algorithm it should be possible to apply this *asymptotic determinism of RSK insertion* also to study the the dynamics of the entries of an insertion tableau. Thus description of the asymptotics of a *single particle* in the insertion tableau (which corresponds to the *pointwise convergence* of a random transformation, in probability) seems to be a low risk task with the chance of success neighboring with certainty.

It would be also interesting to describe the *collective* behavior of the particles (which would correspond to convergence in much stronger topology of, say, *uniform convergence* of a random transformation, in probability); for such a stronger result it is not clear if an application of the *asymptotic determinism of RSK insertion* would be a sufficient tool for such a more ambitious problem.

3.2. **Research goal (B): jeu de taquin.** We continue discussion from Section 2.3.4

The first naive thought would be to apply the results of Romik and PI [RŚ15]. Regretfully, this does not seem to be a good idea, as we shall discuss below. The key observation from [RŚ15] was that a random (infinite) Young tableau with Plancherel distribution can be generated as the recording tableau associated to an infinite word $(x_0, x_1, \dots)$ of i.i.d. random letters with the uniform distribution on the unit interval $[0, 1]$. Then the large-scale asymptotics of jeu de taquin trajectory is governed only by the first entry x_0. If we tried to adapt this concept to the setup of random Young tableaux with a specific rectangular shape we would end up with a random finite word $(x_0, \dots, x_n)$ which is essentially a random *Erdős–Szekeres permutation* [Rom06]. By the results of Romik [Rom06], the first entry x_0 of such a word converges to a deterministic limit. On the other hand, the results of computer simulations (Figure 5) would suggest that jeu de taquin trajectory *does not* converge to a single deterministic limit, but rather to a random curve from some one-dimensional family.

This negative observation indicates that the results of [RŚ15] cannot be applied in our context directly. However, it seems that some general ideas from that work could still be recycled and applied for studying jeu de taquin trajectories for random Young tableaux filling a large class of (random) (skew) Young diagrams. Roughly speaking, the proof of RSK insertion determinism from [RŚ15] was based on investigation of combinatorics of the induction and outer product of representations. Maybe some other operation on representations (some specific version of restriction to a subgroup?) will turn out to be the right one for our new context. The preliminary results seem to be encouraging and this research goal with high probability is within our reach.

Other ingredients of the proof involve known laws of large numbers for random Young diagrams and tableaux [Bia98, Bia01, PR07].

In the most ambitious version of this research goal which involves *skew* Young tableaux, some results about the asymptotics of (reducible) characters of the symmetric groups $\mathfrak{S}_n$ related to *skew Young diagrams* will be necessary; regretfully these results do not seem to be available today [Sta03] and we might need to prove them ourselves. It is not clear if out methods will be sufficient for this most general version.

3.3. **Research goal (C): TASEP with the uniform history distribution.** We continue discussion from Section 2.5.

The Rost's mapping between growth sequences of Young diagrams and time-evolutions of particle systems [Ros81] allows to translate some statements concerning Young tableaux into the language of interacting particle systems. In particular, in our previous paper [RŚ15] we explained the relationship between the jeu de taquin trajectory on one side and the dynamics of a second class particle on the other.

It seems therefore that the results of the Research goal (B), see Section 3.2 above, can be to large extent translated to the language of the current research goal.

3.4. **Research goal (D). Brownian motion limit of Dykema–Haagerup identities.** We continue discussion from Section 2.6

The success of this research goal is based on a deep understaning of a modification of RSK algorithm which appears (in a somewhat concealed way) in the bijective proof of Dykema–Haagerup identities [Śni06b]. Some preliminary promising results in this field were obtained in a collaboration with Artur Jeż (unpublished).

3.5. **Research goal (E). Other problems.** This research project has a quite wide scope of investigating (dynamic) limit shapes of various large random combinatorial structures, for this reason we also plan to attack some other open problems, not discussed in datail in Section 2.

One of specific examples which we have in mind is proving the conjectures concerning the dynamical limit shapes for particles in uniformly random sorting networks [AHRV07]. Our plan is to use Edelman–Greene bijection [EG87] and to reverse-engineer the problem, possibly using the ideas and techniques developed by PI together with Romik [RŚ15]. This part of the research proposal is quite risky and we are not sure if this approach will turn out succesful.

We also plan to explore further applications of the asymptotic analysis of the characters of the linear groups $\mathrm{GL}_d(\mathbb{C})$ [GP15].

4. RESEARCH METHODOLOGY

Our main tools will be **the combinatorial methods and objects related to *the asymptotic representation theory of the symmetric groups***, in particular *the Kerov–Olshanski algebra of polynomial functions on the set of Young diagrams* [KO94], *Jucys–Murphy elements* [CSST14], *Kerov character polynomials* [DFŚ10, Śni13].

We will also use the general methods of **the analytic combinatorics** [FS09], **the algebraic combinatorics** [Sta13] and **the enumerative combinatorics** [Sta97, Sta99], as well as **the combinatorial methods of the random matrix theory** [NS06].

The trajectories of jeu de taquin seem to be examples of anamalous diffusions with rather elusive probabilistic characteristics. For this reason in the initial phase of the project we plan to run a series of computer simulations on large Young diagrams in order to get a heuristic insight, for example on the question about the (non)gaussianity of the shape fluctuations.

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I. ZAŁĄCZNIKI

Poniżej wymienione załączniki są załączone do niniejszego wniosku.

Instytut Matematyczny Polskiej Akademii Nauk

1. Oświadczenie o niewystępowaniu pomocy publicznej wraz z kwestionariuszem.