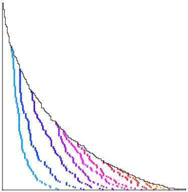
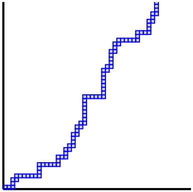
dynamics behind random partitions and random permutations

joint work with Dan Romik

Piotr Śniady





Question: "*how RSK algorithm (applied to random data) reacts to local changes of the input?*"

Robinson–Schensted–Knuth algorithm is a bijection. . .

input:

- word $\mathbf{w} = (w_1, \dots, w_n)$

output:

- semistandard tableau P,
- standard tableau Q,

tableaux P and Q have the same shape with n boxes

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example:

$\mathbf{w} = (23,\ 53,\ 74,\ 16,\ 99,\ 70,\ 82,\ 37,\ 41)$

74	99		
23	53	70	
16	37	41	82

insertion tableau $P(\mathbf{w})$

8	9		
4	6	7	
1	2	3	5

recording tableau $Q(\mathbf{w})$

Robinson-Schensted-Knuth algorithm — induction step

74	99		
23	53	70	
16	37	41	82

insertion tableau $P(\mathbf{w})$

8	9		
4	6	7	
1	2	3	5

recording tableau $Q(\mathbf{w})$

$$\mathbf{w} = (23, 53, 74, 16, 99, 70, 82, 37, 41)$$

Robinson-Schensted-Knuth algorithm — induction step

74	99		
23	53	70	
16	37	41	82

insertion tableau $P(\mathbf{w})$

8	9		
4	6	7	
1	2	3	5

recording tableau $Q(\mathbf{w})$

$\mathbf{w} = (23, 53, 74, 16, 99, 70, 82, 37, 41, 18)$

Robinson-Schensted-Knuth algorithm — induction step

74	99		
23	53	70	
16	37	41	82

18

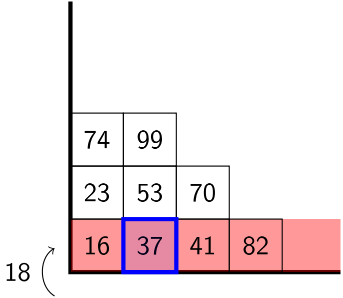
insertion tableau $P(\mathbf{w})$

8	9		
4	6	7	
1	2	3	5

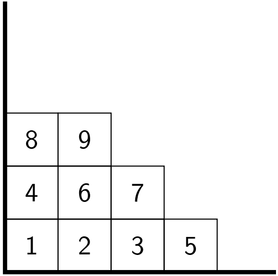
recording tableau $Q(\mathbf{w})$

$\mathbf{w} = (23, 53, 74, 16, 99, 70, 82, 37, 41, 18)$

Robinson-Schensted-Knuth algorithm — induction step



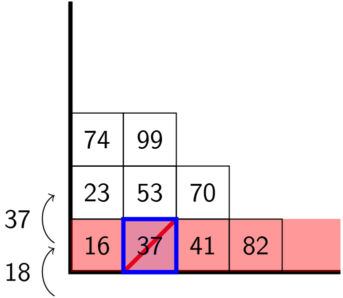
insertion tableau $P(\mathbf{w})$



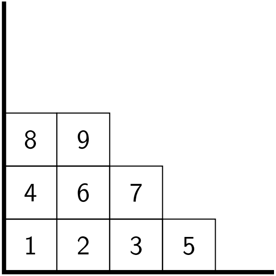
recording tableau $Q(\mathbf{w})$

$\mathbf{w} = (23, 53, 74, 16, 99, 70, 82, 37, 41, 18)$

Robinson-Schensted-Knuth algorithm — induction step



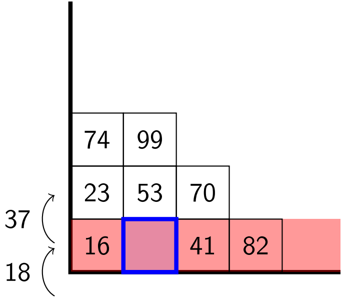
insertion tableau $P(\mathbf{w})$



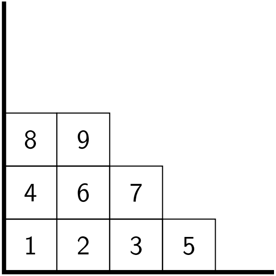
recording tableau $Q(\mathbf{w})$

$\mathbf{w} = (23, 53, 74, 16, 99, 70, 82, 37, 41, 18)$

Robinson-Schensted-Knuth algorithm — induction step



insertion tableau $P(\mathbf{w})$



recording tableau $Q(\mathbf{w})$

$\mathbf{w} = (23, 53, 74, 16, 99, 70, 82, 37, 41, 18)$

Robinson-Schensted-Knuth algorithm — induction step

37

74	99		
23	53	70	
16	18	41	82

insertion tableau $P(\mathbf{w})$

8	9		
4	6	7	
1	2	3	5

recording tableau $Q(\mathbf{w})$

$\mathbf{w} = (23, 53, 74, 16, 99, 70, 82, 37, 41, 18)$

Robinson-Schensted-Knuth algorithm — induction step

74	99		
23	53	70	
16	18	41	82

37

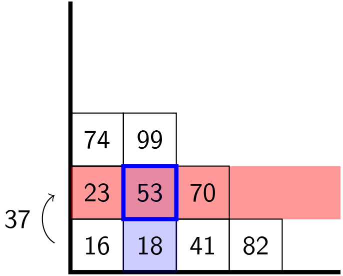
insertion tableau $P(\mathbf{w})$

8	9		
4	6	7	
1	2	3	5

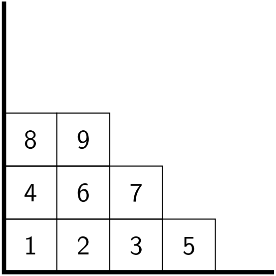
recording tableau $Q(\mathbf{w})$

$\mathbf{w} = (23, 53, 74, 16, 99, 70, 82, 37, 41, 18)$

Robinson-Schensted-Knuth algorithm — induction step



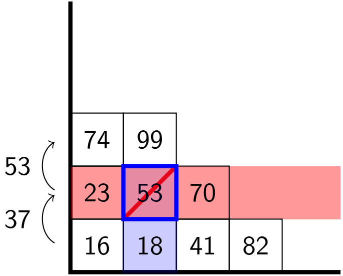
insertion tableau $P(\mathbf{w})$



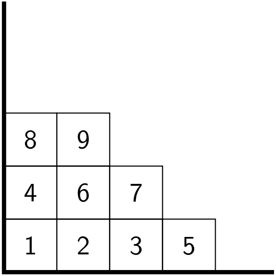
recording tableau $Q(\mathbf{w})$

$\mathbf{w} = (23, 53, 74, 16, 99, 70, 82, 37, 41, 18)$

Robinson-Schensted-Knuth algorithm — induction step



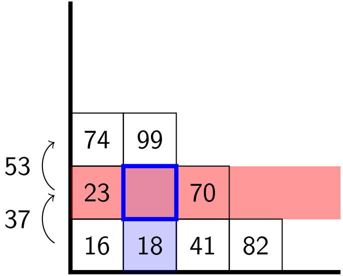
insertion tableau $P(\mathbf{w})$



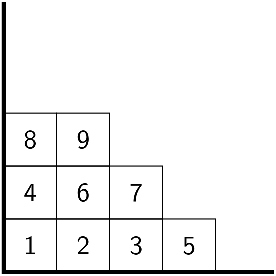
recording tableau $Q(\mathbf{w})$

$\mathbf{w} = (23, 53, 74, 16, 99, 70, 82, 37, 41, 18)$

Robinson-Schensted-Knuth algorithm — induction step



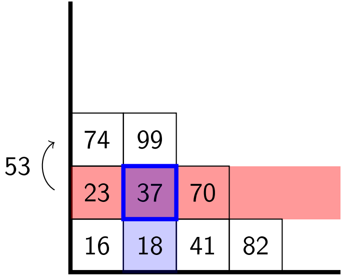
insertion tableau $P(\mathbf{w})$



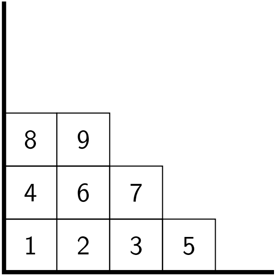
recording tableau $Q(\mathbf{w})$

$\mathbf{w} = (23, 53, 74, 16, 99, 70, 82, 37, 41, 18)$

Robinson-Schensted-Knuth algorithm — induction step



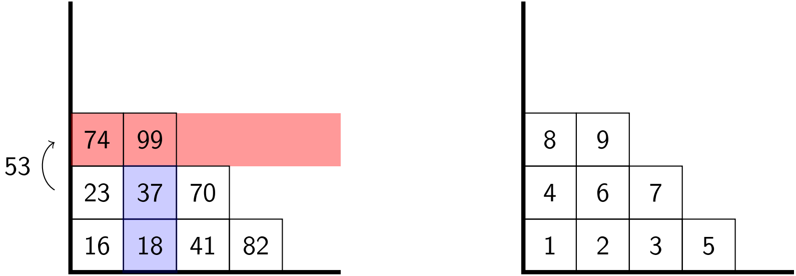
insertion tableau $P(\mathbf{w})$



recording tableau $Q(\mathbf{w})$

$\mathbf{w} = (23, 53, 74, 16, 99, 70, 82, 37, 41, 18)$

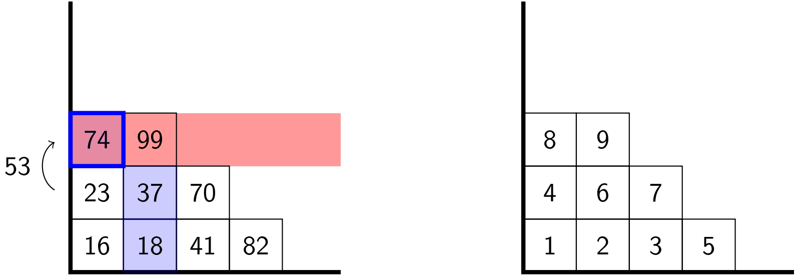
Robinson-Schensted-Knuth algorithm — induction step



insertion tableau $P(\mathbf{w})$ recording tableau $Q(\mathbf{w})$

$\mathbf{w} = (23, 53, 74, 16, 99, 70, 82, 37, 41, {\color{blue}18})$

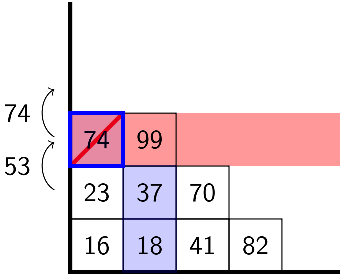
Robinson-Schensted-Knuth algorithm — induction step



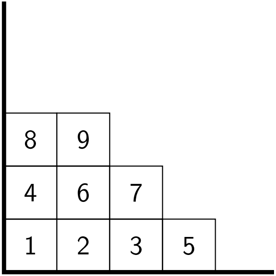
insertion tableau $P(\mathbf{w})$ recording tableau $Q(\mathbf{w})$

$\mathbf{w} = (23, 53, 74, 16, 99, 70, 82, 37, 41, 18)$

Robinson-Schensted-Knuth algorithm — induction step



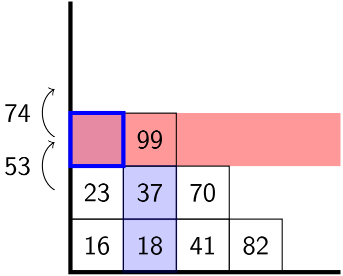
insertion tableau $P(\mathbf{w})$



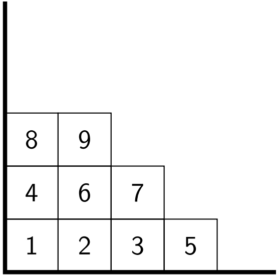
recording tableau $Q(\mathbf{w})$

$\mathbf{w} = (23, 53, 74, 16, 99, 70, 82, 37, 41, 18)$

Robinson-Schensted-Knuth algorithm — induction step



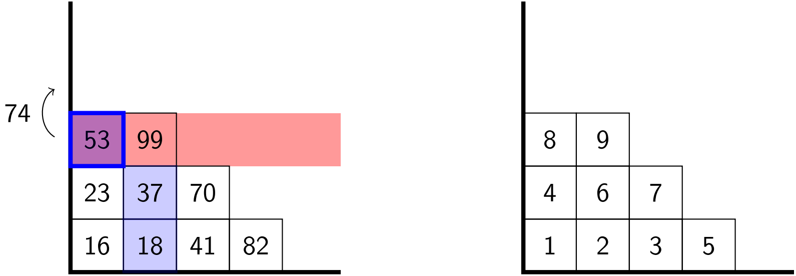
insertion tableau $P(\mathbf{w})$



recording tableau $Q(\mathbf{w})$

$\mathbf{w} = (23, 53, 74, 16, 99, 70, 82, 37, 41, 18)$

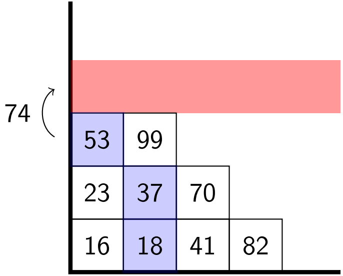
Robinson-Schensted-Knuth algorithm — induction step

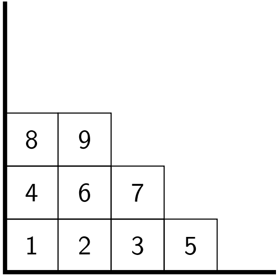


insertion tableau $P(\mathbf{w})$ recording tableau $Q(\mathbf{w})$

$\mathbf{w} = (23, 53, 74, 16, 99, 70, 82, 37, 41, 18)$

Robinson-Schensted-Knuth algorithm — induction step

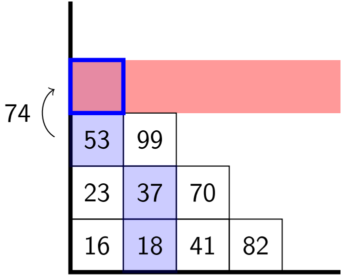




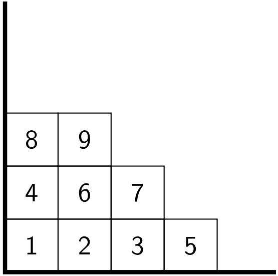
insertion tableau $P(\mathbf{w})$ recording tableau $Q(\mathbf{w})$

$\mathbf{w} = (23, 53, 74, 16, 99, 70, 82, 37, 41, 18)$

Robinson-Schensted-Knuth algorithm — induction step



insertion tableau $P(\mathbf{w})$



recording tableau $Q(\mathbf{w})$

$\mathbf{w} = (23, 53, 74, 16, 99, 70, 82, 37, 41, 18)$

Robinson-Schensted-Knuth algorithm — induction step

74			
53	99		
23	37	70	
16	18	41	82

insertion tableau $P(\mathbf{w})$

8	9		
4	6	7	
1	2	3	5

recording tableau $Q(\mathbf{w})$

$\mathbf{w} = (23, 53, 74, 16, 99, 70, 82, 37, 41, 18)$

Robinson-Schensted-Knuth algorithm — induction step

74			
53	99		
23	37	70	
16	18	41	82

insertion tableau $P(\mathbf{w})$

8	9		
4	6	7	
1	2	3	5

recording tableau $Q(\mathbf{w})$

$\mathbf{w} = (23, 53, 74, 16, 99, 70, 82, 37, 41, 18)$

Robinson-Schensted-Knuth algorithm — induction step

74			
53	99		
23	37	70	
16	18	41	82

insertion tableau $P(\mathbf{w})$

new box

$\boxed{10}$			
8	9		
4	6	7	
1	2	3	5

recording tableau $Q(\mathbf{w})$

$\mathbf{w} = (23, 53, 74, 16, 99, 70, 82, 37, 41, 18)$

Robinson-Schensted-Knuth algorithm — induction step

bumping route

74			
53	99		
23	37	70	
16	18	41	82

insertion tableau $P(\mathbf{w})$

new box

10			
8	9		
4	6	7	
1	2	3	5

recording tableau $Q(\mathbf{w})$

$\mathbf{w} = (23, 53, 74, 16, 99, 70, 82, 37, 41, 18)$

Robinson-Schensted-Knuth algorithm — induction step

74			
53	99		
23	37	70	
16	18	41	82

insertion tableau $P(\mathbf{w})$

10			
8	9		
4	6	7	
1	2	3	5

recording tableau $Q(\mathbf{w})$

$$\mathbf{w} = (23, 53, 74, 16, 99, 70, 82, 37, 41, 18)$$

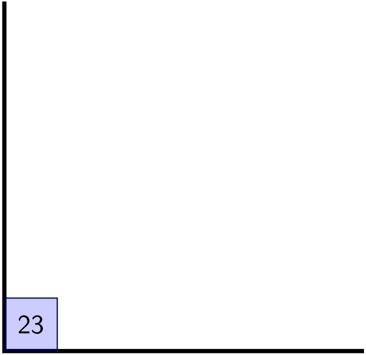
Robinson-Schensted-Knuth algorithm

insertion tableau $P(\mathbf{w})$

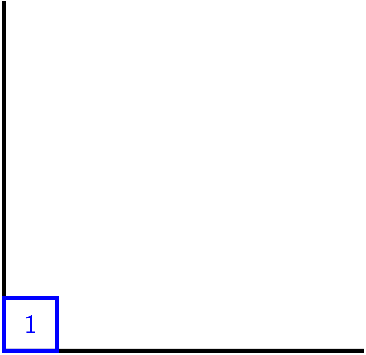
recording tableau $Q(\mathbf{w})$

$\mathbf{w} = \emptyset$

Robinson-Schensted-Knuth algorithm



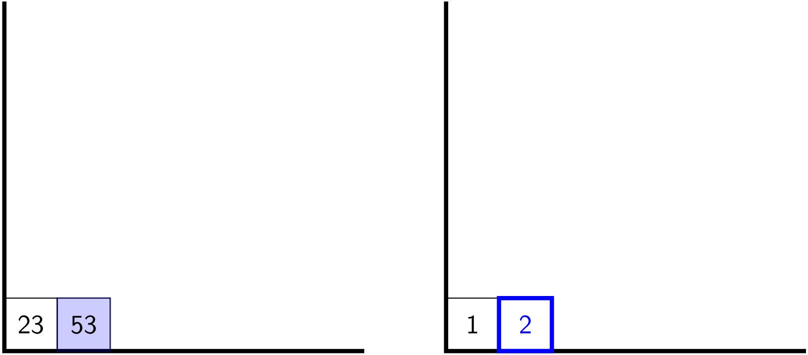
insertion tableau $P(\mathbf{w})$



recording tableau $Q(\mathbf{w})$

$\mathbf{w} = (23)$

Robinson-Schensted-Knuth algorithm



insertion tableau $P(\mathbf{w})$

recording tableau $Q(\mathbf{w})$

$\mathbf{w} = (23, 53)$

Robinson-Schensted-Knuth algorithm

23	53	74

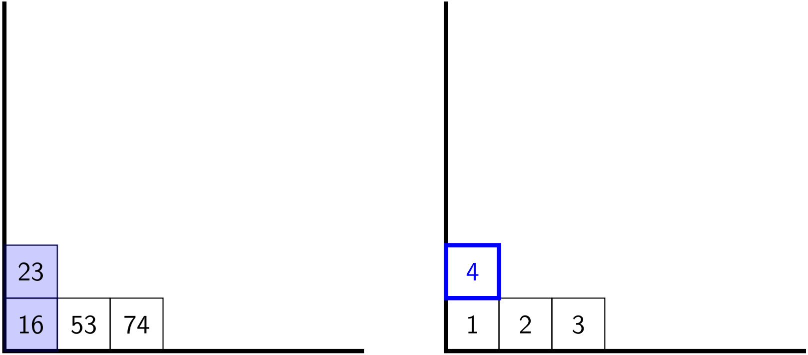
insertion tableau $P(\mathbf{w})$

1	2	3

recording tableau $Q(\mathbf{w})$

$\mathbf{w} = (23,\ 53,\ 74)$

Robinson-Schensted-Knuth algorithm



insertion tableau $P(\mathbf{w})$ recording tableau $Q(\mathbf{w})$

$\mathbf{w} = (23,\ 53,\ 74,\ {\color{blue}16})$

Robinson-Schensted-Knuth algorithm

23			
16	53	74	99

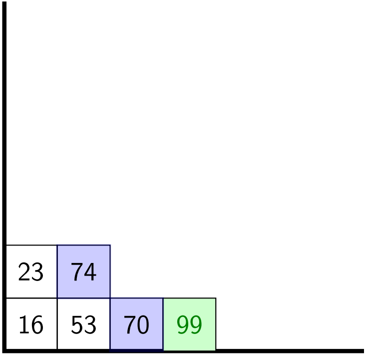
insertion tableau $P(\mathbf{w})$

4			
1	2	3	$\boxed{5}$

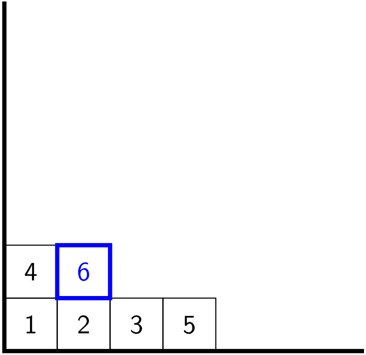
recording tableau $Q(\mathbf{w})$

$$\mathbf{w} = (23,\ 53,\ 74,\ 16,\ 99)$$

Robinson-Schensted-Knuth algorithm



insertion tableau $P(\mathbf{w})$



recording tableau $Q(\mathbf{w})$

$\mathbf{w} = (23, 53, 74, 16, 99, 70)$

Robinson-Schensted-Knuth algorithm

23	74	99	
16	53	70	82

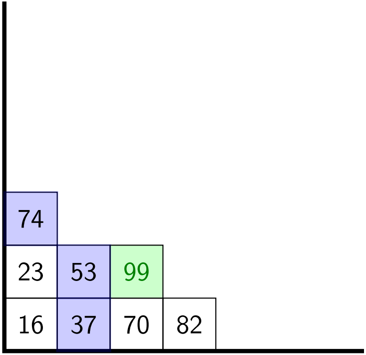
insertion tableau $P(\mathbf{w})$

4	6	$\boxed{7}$	
1	2	3	5

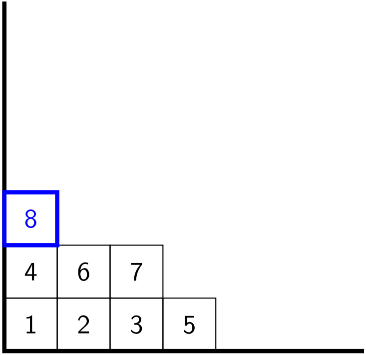
recording tableau $Q(\mathbf{w})$

$$\mathbf{w} = (23, 53, 74, 16, 99, 70, 82)$$

Robinson-Schensted-Knuth algorithm



insertion tableau $P(\mathbf{w})$



recording tableau $Q(\mathbf{w})$

$\mathbf{w} = (23, 53, 74, 16, 99, 70, 82, 37)$

Robinson-Schensted-Knuth algorithm

74	99		
23	53	70	
16	37	41	82

insertion tableau $P(\mathbf{w})$

8	9		
4	6	7	
1	2	3	5

recording tableau $Q(\mathbf{w})$

$\mathbf{w} = (23, 53, 74, 16, 99, 70, 82, 37, 41)$

Robinson-Schensted-Knuth algorithm

74			
53	99		
23	37	70	
16	34	41	82

insertion tableau $P(\mathbf{w})$

10			
8	9		
4	6	7	
1	2	3	5

recording tableau $Q(\mathbf{w})$

$\mathbf{w} = (23, 53, 74, 16, 99, 70, 82, 37, 41, 34)$

Robinson-Schensted-Knuth algorithm

74			
53	99		
23	37	70	82
16	34	41	73

insertion tableau $P(\mathbf{w})$

10			
8	9		
4	6	7	$\boxed{11}$
1	2	3	5

recording tableau $Q(\mathbf{w})$

$\mathbf{w} = (23, 53, 74, 16, 99, 70, 82, 37, 41, 34, 73)$

Robinson-Schensted-Knuth algorithm

74			
53			
23	99		
16	37	70	82
2	34	41	73

insertion tableau $P(\mathbf{w})$

12			
10			
8	9		
4	6	7	11
1	2	3	5

recording tableau $Q(\mathbf{w})$

$$\mathbf{w} = (23, 53, 74, 16, 99, 70, 82, 37, 41, 34, 73, 2)$$

Robinson-Schensted-Knuth algorithm

74			
53	99		
23	37		
16	34	70	82
2	24	41	73

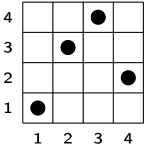
insertion tableau $P(\mathbf{w})$

12			
10	$\boxed{13}$		
8	9		
4	6	7	11
1	2	3	5

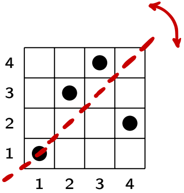
recording tableau $Q(\mathbf{w})$

$$\mathbf{w} = (23, 53, 74, 16, 99, 70, 82, 37, 41, 34, 73, 2, 24)$$

magic symmetries of RSK

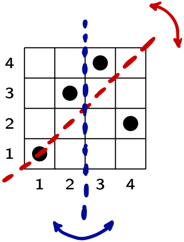
$(1, 3, 4, 2) =$  $\mapsto \left(\begin{ytableau} 3 \\ 1 & 2 & 4 \end{ytableau} \; , \; \begin{ytableau} 4 \\ 1 & 2 & 3 \end{ytableau} \right)$

magic symmetries of RSK

$(1,3,4,2) =$ 

$$\mapsto \left(\begin{array}{|c|cc} \hline 3 & \multicolumn{2}{c}{} \\ \hline 1 & \multicolumn{1}{|c|}{2} & \multicolumn{1}{c|}{4} \\ \hline \end{array} \;,\; \begin{array}{|c|cc} \hline 4 & \multicolumn{2}{c}{} \\ \hline 1 & \multicolumn{1}{|c|}{2} & \multicolumn{1}{c|}{3} \\ \hline \end{array} \right)$$

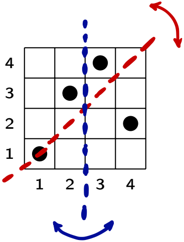
magic symmetries of RSK

$(1,3,4,2) =$ 

$$\mapsto \left(\begin{array}{|c|cc} \hline 3 & \multicolumn{2}{c}{} \\ \hline 1 & \multicolumn{1}{|c|}{2} & \multicolumn{1}{c|}{4} \\ \hline \end{array} \;,\; \begin{array}{|c|cc} \hline 4 & \multicolumn{2}{c}{} \\ \hline 1 & \multicolumn{1}{|c|}{2} & \multicolumn{1}{c|}{3} \\ \hline \end{array} \right)$$

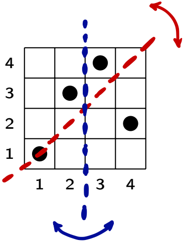
magic symmetries of RSK

$(1,3,4,2) =$



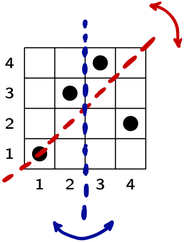
$$\mapsto \left(\begin{array}{|c|cc} \hline 3 & \multicolumn{2}{c}{} \\ \hline 1 & \multicolumn{1}{|c|}{2} & \multicolumn{1}{c|}{4} \\ \hline \end{array} \; , \; \begin{array}{|c|cc} \hline 4 & \multicolumn{2}{c}{} \\ \hline 1 & \multicolumn{1}{|c|}{2} & \multicolumn{1}{c|}{3} \\ \hline \end{array} \right)$$

magic symmetries of RSK

$(1, 3, 4, 2) =$ 

$$\mapsto \left(\begin{array}{|c|cc} \hline 3 & \multicolumn{2}{c}{} \\ \hline 1 & 2 & \multicolumn{1}{c|}{4} \\ \hline \end{array} \; , \; \begin{array}{|c|cc} \hline 4 & \multicolumn{2}{c}{} \\ \hline 1 & 2 & \multicolumn{1}{c|}{3} \\ \hline \end{array} \right)$$

magic symmetries of RSK

$(1,3,4,2) =$ 

$\mapsto \left(\begin{array}{|c|cc} \hline 3 & \multicolumn{2}{c}{} \\ \hline 1 & \multicolumn{1}{|c|}{2} & \multicolumn{1}{c|}{4} \\ \hline \end{array} \;,\; \begin{array}{|c|cc} \hline 4 & \multicolumn{2}{c}{} \\ \hline 1 & \multicolumn{1}{|c|}{2} & \multicolumn{1}{c|}{3} \\ \hline \end{array} \right)$

180°

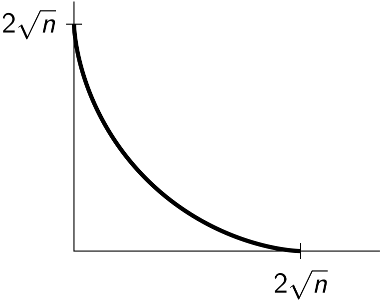
180°

old problem: RSK applied to random input

if $\mathbf{w} = (w_1, \dots, w_n)$ is i.i.d. uniform $U(0,1)$ then
the common shape of $P(\mathbf{w})$ and $Q(\mathbf{w})$

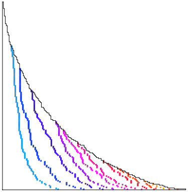
(Plancherel measure on Young diagrams with n boxes)

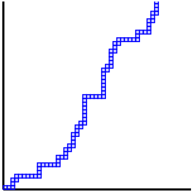
converges for $n \to \infty$
to some limit shape



$\longrightarrow$ Logan&Shepp, Vershik&Kerov 1977

dynamic problems:
what happens if we **modify** the random input of RSK?



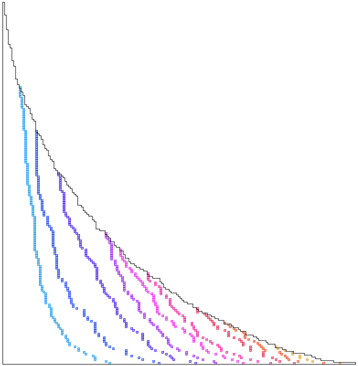


what happens to the insertion tableau $P(\mathbf{w})$ if we append one letter at the end?

what happens to the insertion tableau $P(\mathbf{w})$ if we append a lot of letters at the end?

what happens to the recording tableau $Q(\mathbf{w})$ if we remove the first letter from $\mathbf{w}$?

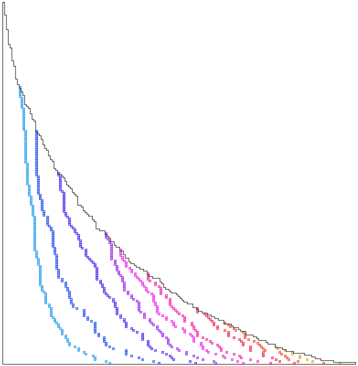
bumping routes



problem ⟶ MOORE 2006

what can we say about the shapes of the bumping routes?

bumping routes

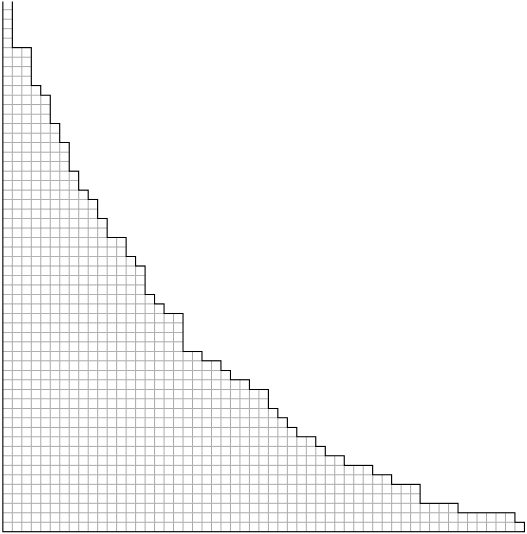


problem $\longrightarrow$ MOORE 2006

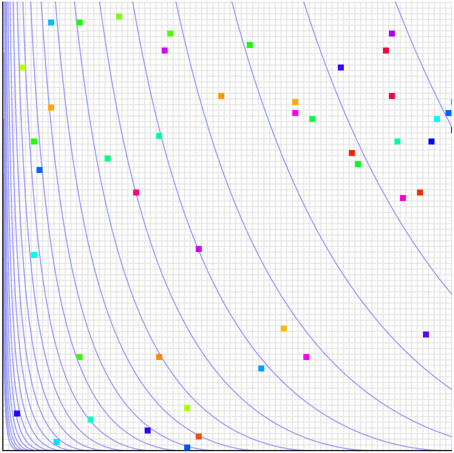
what can we say about the shapes of the bumping routes?

if $\mathbf{w} = (w_1, \ldots, w_n)$ are i.i.d. uniform $U(0, 1)$,
the rescaled bumping route
obtained by adding a new entry w_{n+1}
converges in probability (as $n \to \infty$) to a deterministic curve

diffusion of a box in the insertion tableau $P(\mathbf{w})$



hydrodynamics of the insertion tableau $P(\mathbf{w})$



$m = 1.6 \cdot 10^4$

8	13	18	32
6	9	12	23
4	5	7	19
1	2	3	10

jeu de taquin

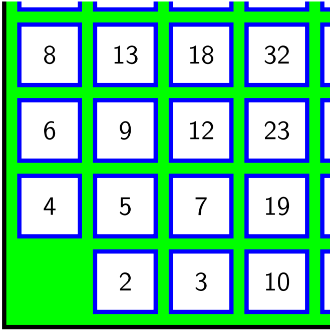
start with (infinite) tableau
$t = Q(w_1, w_2, \ldots),$

8	13	18	32
6	9	12	23
4	5	7	19
1	2	3	10

jeu de taquin

start with (infinite) tableau $t = Q(w_1, w_2, \ldots)$,

① remove corner box,



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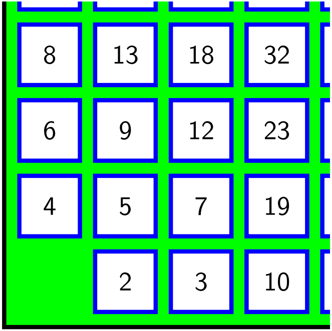
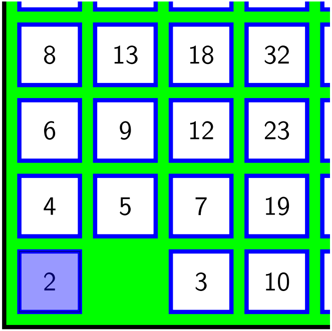


jeu de taquin

start with (infinite) tableau
$t = Q(w_1, w_2, \dots)$,

① remove corner box,
② sliding,



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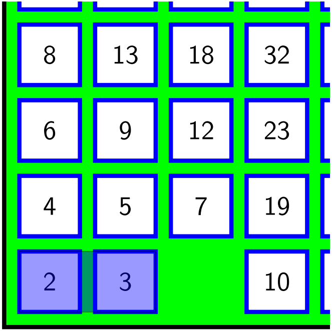

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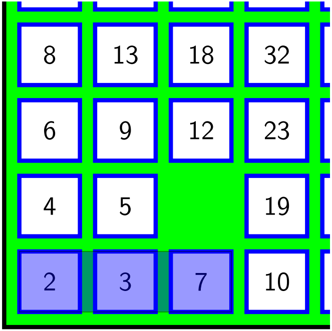

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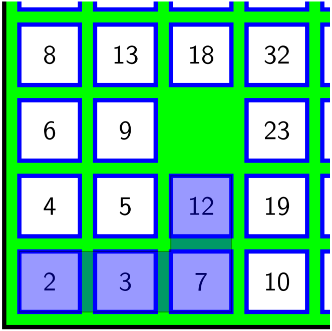

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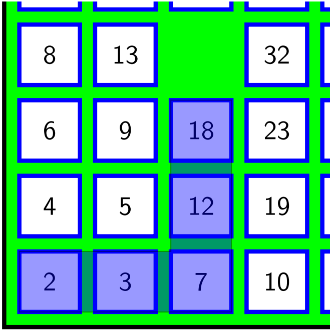

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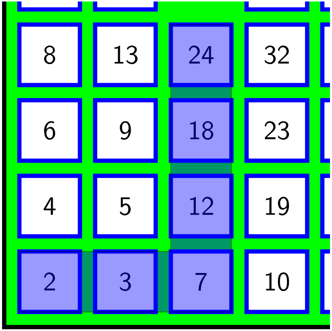

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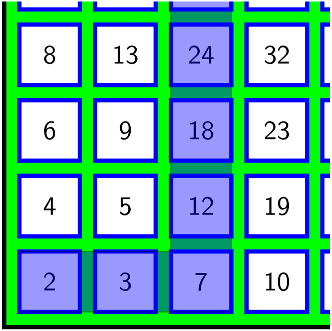

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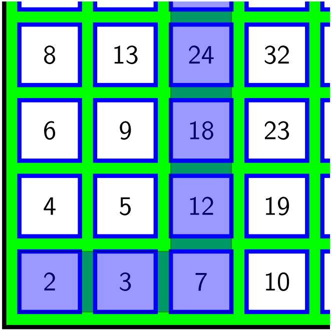
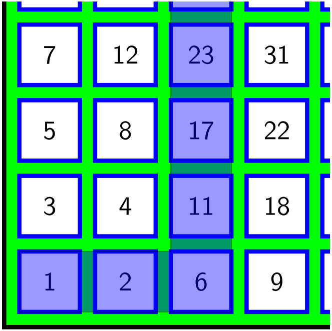


jeu de taquin

start with (infinite) tableau $t = Q(w_1, w_2, \dots)$,

① remove corner box,
② sliding,
③ subtract 1 from all boxes



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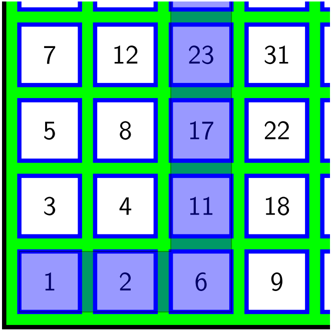


jeu de taquin

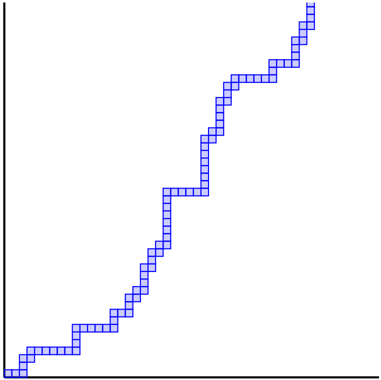
start with (infinite) tableau $t = Q(w_1, w_2, \dots)$,

① remove corner box,
② sliding,
③ subtract 1 from all boxes

output:

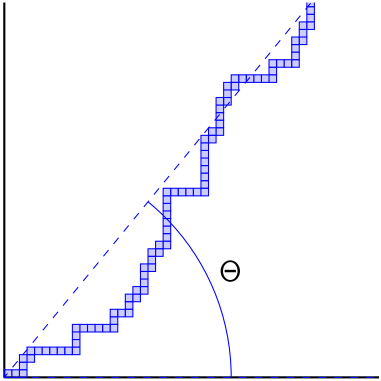
- new tableau $= Q(\cancel{w_1}, w_2, w_3, \dots)$,
- blue trajectory $\mathbf{c}(t) = (c_1, c_2, \dots)$

trajectories of jeu de taquin



if $t = Q(w_1, w_2, \dots)$ is random, Plancherel distributed. . .

trajectories of jeu de taquin



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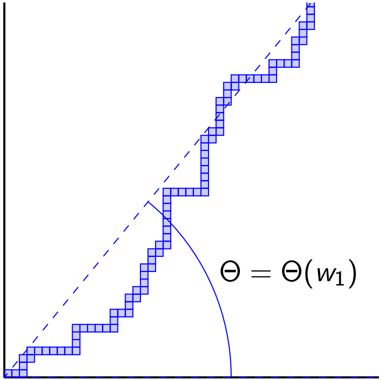
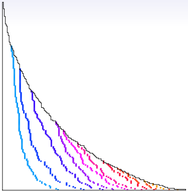


if $t = Q(w_1, w_2, \dots)$ is random, Plancherel distributed...

then its jdt trajectory $\mathbf{c}(t)$
is (almost surely) asymptotically
a straight line
which depends on w_1, i.e.

$$\lim_{k\to\infty} \frac{c_k}{\|c_k\|} = \Big(\cos\Theta(w_1), \sin\Theta(w_1)\Big)$$



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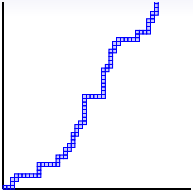


what happens to the recording tableau $Q(\mathbf{w})$ if we remove the first letter from $\mathbf{w}$?

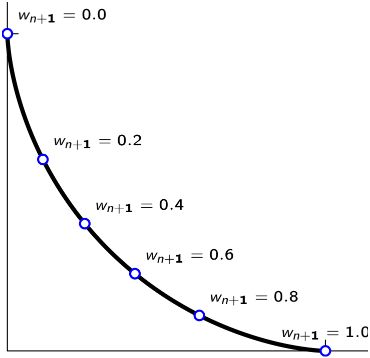
exercise

use magic symmetries of RSK
to show that a solution to any of these problems
gives solutions to the other ones

the key problem

let $\mathbf{w} = (w_1, \dots, w_n, w_{n+1})$,
with $w_1, \dots, w_n$ random, iid $U(0,1)$
and w_{n+1} deterministic

$\square$ is the box containing $n+1$
in the recording tableau $Q(\mathbf{w})$...



the key theorem, Romik&Śniady 2015

$$\left\| \frac{\square}{\sqrt{n}} - \left(\mathrm{RSKcos}\, w_{n+1}, \mathrm{RSKsin}\, w_{n+1}\right) \right\| \xrightarrow[\text{in probability}]{n\to\infty} 0$$

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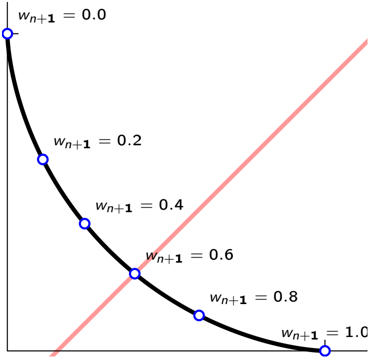

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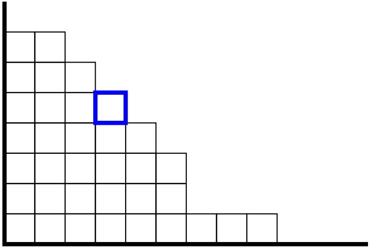
reduction of the problem: adding randomness

instead of (for deterministic w_{n+1})...

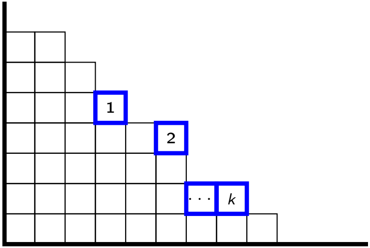
$$Q(w_1, \dots, w_n, \; w_{n+1} \qquad) \setminus Q(w_1, \dots, w_n) = \{ \; \square \; \}$$



reduction of the problem: adding randomness

we study (for random $0 < t_1 < \cdots < t_k < 1$)

$$Q(w_1, \ldots, w_n,\ t_1, \ldots, t_k) \setminus Q(w_1, \ldots, w_n) = \left\{ \boxed{1}, \ldots, \boxed{k} \right\}$$



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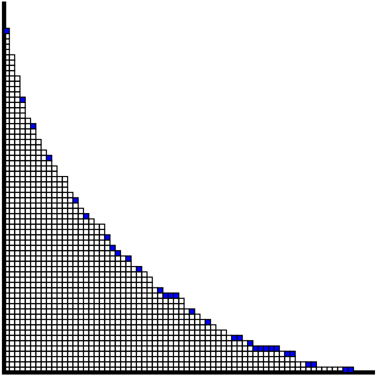


representations of the symmetric groups

irreducible representation ρ^λ of the symmetric group $\mathfrak{S}_n$ $\longleftrightarrow$ Young diagram λ with n boxes

---

random irreducible component of a reducible representation V:

$$\mathbb{P}(\lambda) = \frac{(\text{multiplicity of } \lambda \text{ in } V) \cdot (\text{dimension of } \rho^\lambda)}{\text{dimension of } V}$$

plactic Littlewood–Richardson rule

if $0 \leq w_1, \dots, w_n \leq 1$ is a random sequence, such that

$$\text{shape of } P(w_1, \dots, w_n) = \lambda;$$

and $0 \leq t_1, \dots, t_k \leq 1$ is a random sequence, such that

$$\text{shape of } P(t_1, \dots, t_k) = \mu$$

then the random Young diagram

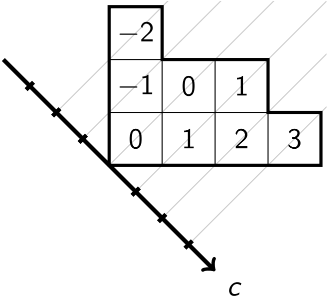
$$\text{shape of } P(w_1, \dots, w_n, t_1, \dots, t_k)$$

has the same distribution as random irreducible component of

$$\rho^\lambda \otimes \rho^\mu \big\uparrow_{\mathfrak{S}_n \times \mathfrak{S}_k}^{\mathfrak{S}_{n+k}}$$

content of a box

$c(\square) = (x\text{-coordinate}) - (y\text{-coordinate})$



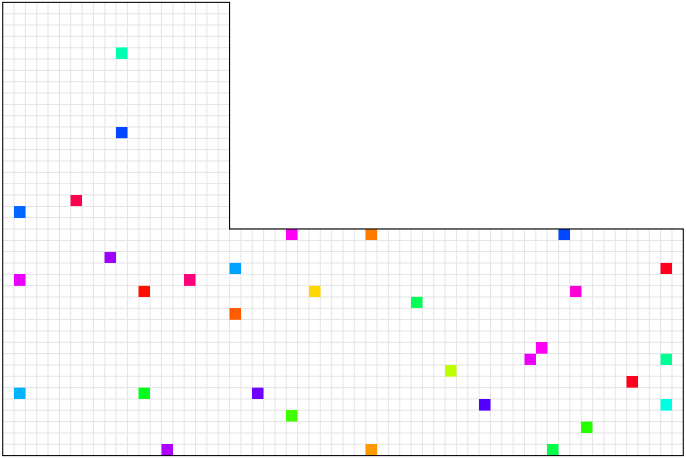
$\longrightarrow$Algimantas Adolfas Jucys

Jucys–Murphy elements

$$X_i = (1,i)+(2,i)+\cdots+(i-1,i) \quad \text{for } i \in \{1,\dots,n\}$$

$X_1,\dots,X_n$ are elements of the symmetric group algebra $\mathbb{C}[\mathfrak{S}_n]$

new! removing independence of the letters $w_1, w_2, \ldots$



$\longrightarrow$TASEP interacting particle systems, second class particles

Dan Romik, Piotr Śniady
Jeu de taquin dynamics on infinite Young tableaux and second class particles
Ann. Probab, Volume 43, Number 2 (2015), 682-737

Piotr Śniady.
Robinson-Schensted-Knuth algorithm, jeu de taquin and Kerov-Vershik measures on infinite tableaux.
SIAM J. Discrete Math. 28 (2014), no. 2, 598-630.

Dan Romik, Piotr Śniady.
Limit shapes of bumping routes in the Robinson-Schensted correspondence.
Random Structures Algorithms 48 (2016), no. 1, 171-182