Museum of visual ART

references, extra notes, exercises

Piotr Śniady

random Young diagrams

Philippe Biane.
Free cumulants and representations of large symmetric groups.
XIIIth International Congress on Mathematical Physics (London, 2000), 321-–326, Int. Press, Boston, MA, 2001.
http://igm.univ-mlv.fr/~biane/ICMP.pdf

Philippe Biane.
Approximate factorization and concentration for characters of symmetric groups.
Internat. Math. Res. Notices 2001, no. 4, 179—192.
https://doi.org/10.48550/arXiv.math/0006111

random Young diagrams

exercise

run the following experiment in SageMath:
let $w = (w_1, \ldots, w_n)$ be a very long random sequence of length $n = 10^4$;

entries are independent random variables,
sampled from $\{1, \ldots, d\}$ with $d = 10^2$

visualize the insertion tableau $P(w)$

visualize the insertion tableau $Q(w)$

now find the theoretical explanation

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surprise: this result never appeared in the literature!

Stanley–Féray character formula, Kerov positivity conjecture

short reading

- Piotr Śniady.
  Stanley character polynomials.
  The mathematical legacy of Richard P. Stanley, 323–334, Amer. Math. Soc., Providence, RI, 2016
  https://doi.org/10.48550/arXiv.1409.7533

- Piotr Śniady.
  Combinatorics of asymptotic representation theory.
  European Congress of Mathematics, 531-–545, Eur. Math. Soc., Zürich, 2013
  https://doi.org/10.48550/arXiv.1203.6509

Stanley–Féray character formula, Kerov positivity conjecture

Maciej Dołęga, Valentin Féray, Piotr Śniady.
Characters of symmetric groups in terms of free cumulants and Frobenius coordinates.
21st International Conference on Formal Power Series and Algebraic Combinatorics (FPSAC 2009), 337–348, Discrete Math. Theor. Comput. Sci. Proc., AK, Assoc. Discrete Math. Theor. Comput. Sci., Nancy, 2009.
https://doi.org/10.48550/arXiv.1105.2549

long reading

Pierre-Loïc Méliot.
Representation theory of symmetric groups.
Discrete Mathematics and its Applications (Boca Raton). CRC Press, Boca Raton, FL, 2017.
https://doi.org/10.1201/9781315371016

bumping routes

- Dan Romik, Piotr Śniady.
  Limit shapes of bumping routes in the Robinson–Schensted correspondence.
  Random Structures Algorithms 48 (2016), no. 1, 171–182
  https://doi.org/10.48550/arXiv.1304.7589

- Mikołaj Marciniak, Łukasz Maślanka, Piotr Śniady.
  Poisson limit of bumping routes in the Robinson–Schensted correspondence.
  Probab. Theory Related Fields 181 (2021), no. 4, 1053–1103
  https://doi.org/10.1007/s00440-021-01084-y

- Mikołaj Marciniak.
  Hydrodynamic limit of the Robinson–Schensted–Knuth algorithm.
  Random Structures Algorithms 60 (2022), no. 1, 106—116
  https://doi.org/10.48550/arXiv.2005.03147

jeu de taquin

- Dan Romik, Piotr Śniady.
  Jeu de taquin dynamics on infinite Young tableaux and second class particles
  Ann. Probab, Volume 43, Number 2 (2015), 682-737
  https://doi.org/10.1214/13-AOP873

- Łukasz Maślanka, Piotr Śniady.
  Limit shapes of evacuation and jeu de taquin paths in random square tableaux.
  Sém. Lothar. Combin. 84B (2020), Art. 8, 12 pp.
  https://www.mat.univie.ac.at/~slc/wpapers/FPSAC2020/8.html

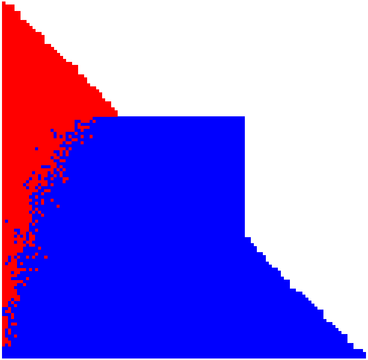
- Łukasz Maślanka, Piotr Śniady.
  Second class particles and limit shapes of evacuation and sliding paths for random tableaux.
  https://doi.org/10.48550/arXiv.1911.08143

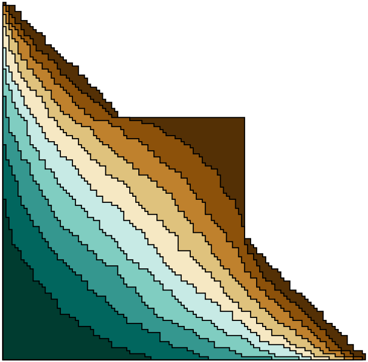
homework: rectification of large random tableaux

computer experiment with SageMath:
the following code generates a random skew tableau with a shape which consists of two squares, touching in the corner;
the ‘red’ small square is in north-west, the ‘blue’ big square is in south-east;
then we rectify the the tableau

```
import random
sizeA=40
sizeB=80
mylist = [0]* sizeA^2 + [1]*sizeB^2
random.shuffle(mylist)
indicesA=[ index+1 for index,entry in enumerate(
    mylist) if entry==0]
indicesB=[ index+1 for index,entry in enumerate(
    mylist) if entry==1]
tabA =StandardTableaux([sizeA]*sizeA).
    random_element()
tabB =StandardTableaux([sizeB]*sizeB).
    random_element()
newtabA= [ [ indicesA[entry-1] for entry in row ]
    for row in tabA]
newtabB= [ [None] * sizeA + [ indicesB[entry-1]
    for entry in row ] for row in tabB]
skew=SkewTableau(newtabB+newtabA)
rectified=skew.rectify()
```

the following two pictures visualize the outcome of the rectification;
the left picture shows the which boxes of the rectified tableau originate from the red and which from the blue tableau
the right picture shows the usual level curves for the rectified tableau





easy homework: find theoretical explanation for the right picture
hard problem: find theoretical explanation for the left picture
what other questions we can ask?