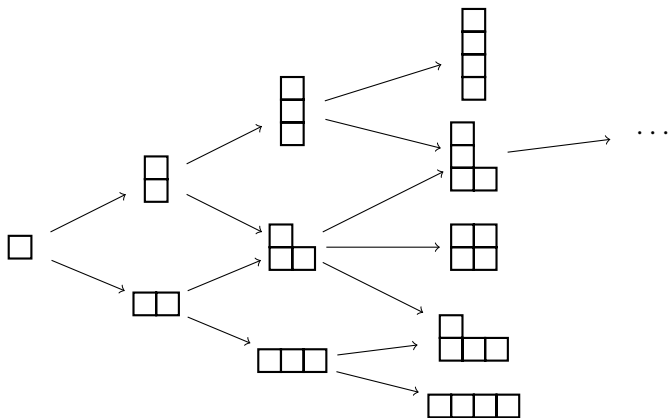


Trajectories of jeu-de-taquin
and representations of the infinite symmetric group
joint work with Dan Romik

Piotr Śniady

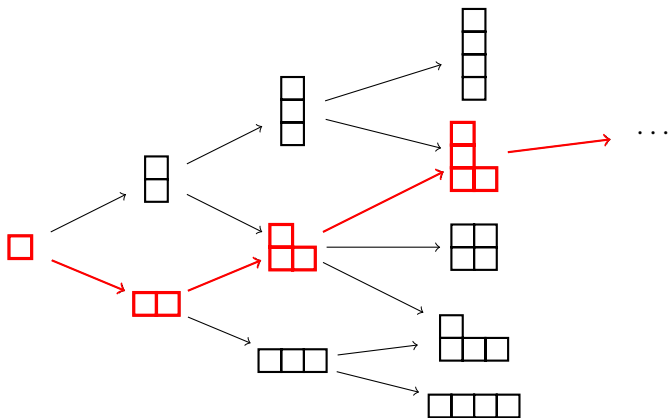
Polish Academy of Sciences
and
University of Wrocław

Young graph: irreducible representations
of symmetric groups $S_1 \subset S_2 \subset S_3 \subset \dots$



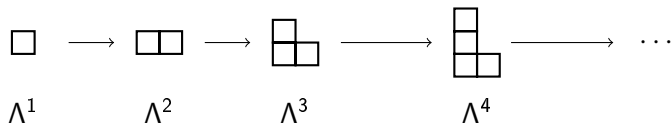
Tool for studying S_∞

Young graph: irreducible representations
of symmetric groups $S_1 \subset S_2 \subset S_3 \subset \dots$

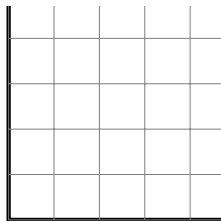


Tool for studying S_∞

paths in Young graph \leftrightarrow tableaux



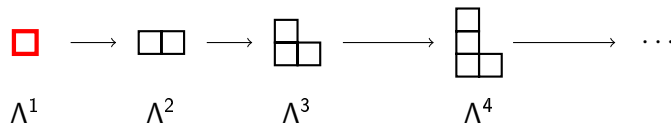
infinite path in Young graph



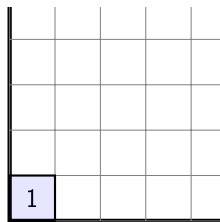
infinite tableau

Ω - set of infinite tableaux / set of infinite paths

paths in Young graph \leftrightarrow tableaux



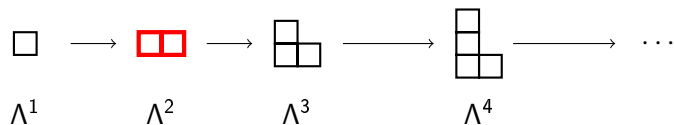
infinite path in Young graph



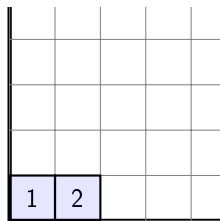
infinite tableau

Ω - set of infinite tableaux / set of infinite paths

paths in Young graph \leftrightarrow tableaux



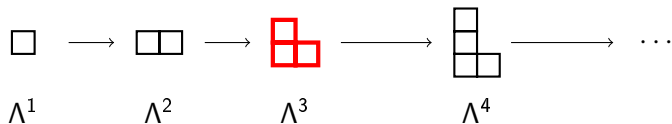
infinite path in Young graph



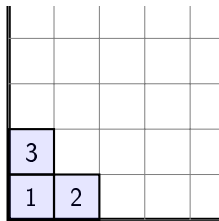
infinite tableau

Ω - set of infinite tableaux / set of infinite paths

paths in Young graph \leftrightarrow tableaux



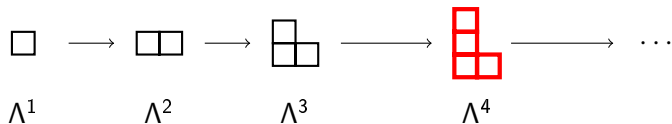
infinite path in Young graph



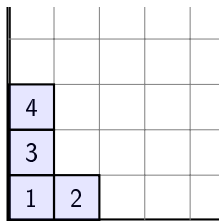
infinite tableau

Ω - set of infinite tableaux / set of infinite paths

paths in Young graph \leftrightarrow tableaux



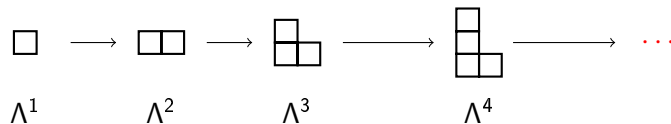
infinite path in Young graph



infinite tableau

Ω - set of infinite tableaux / set of infinite paths

paths in Young graph \leftrightarrow tableaux



infinite path in Young graph

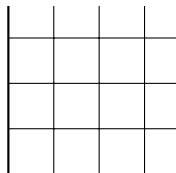
\vdots		\vdots		
6	15	21	24	
4	12	17	19	\dots
3	5	8	11	
1	2	7	9	\dots

infinite tableau

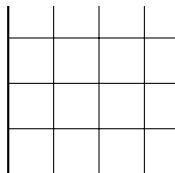
Ω - set of infinite tableaux / set of infinite paths

infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\xrightarrow{\text{RSK}}$ recording tableau



insertion tableau



recording tableau

F O X D R P B Z U L G E A T W N S M Y V C J H Q I K

infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\xrightarrow{\text{RSK}}$ recording tableau

insertion tableau

recording tableau

F O X D R P B Z U L G E A T W N S M Y V C J H Q I K

① *start from the first row,*

infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\xrightarrow{\text{RSK}}$ recording tableau

insertion tableau

recording tableau

F O X D R P B Z U L G E A T W N S M Y V C J H Q I K

- ① *start from the first row,*
- ② *insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,*

infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\xrightarrow{\text{RSK}}$ recording tableau

F			

insertion tableau

recording tableau

F O X D R P B Z U L G E A T W N S M Y V C J H Q I K

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infinite word $\xrightarrow{\text{RSK}}$ recording tableau

F			

insertion tableau

1			

recording tableau

F O X D R P B Z U L G E A T W N S M Y V C J H Q I K

- ① *start from the first row,*
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infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\xrightarrow{\text{RSK}}$ recording tableau

F			

insertion tableau

1			

recording tableau

F O X D R P B Z U L G E A T W N S M Y V C J H Q I K

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F			

insertion tableau

1			

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F			

insertion tableau

1			

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infinite word $\xrightarrow{\text{RSK}}$ recording tableau

F	O		

insertion tableau

1			

recording tableau

F O X D R P B Z U L G E A T W N S M Y V C J H Q I K

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infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\xrightarrow{\text{RSK}}$ recording tableau

F	O		

insertion tableau

1	2		

recording tableau

F O X D R P B Z U L G E A T W N S M Y V C J H Q I K

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infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\xrightarrow{\text{RSK}}$ recording tableau

F	O		

insertion tableau

1	2		

recording tableau

F O X D R P B Z U L G E A T W N S M Y V C J H Q I K

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infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\xrightarrow{\text{RSK}}$ recording tableau

F	O		

insertion tableau

1	2		

recording tableau

F O **X** D R P B Z U L G E A T W N S M Y V C J H Q I K

- ① *start from the first row,*
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infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\xrightarrow{\text{RSK}}$ recording tableau

F	O		

insertion tableau

1	2		

recording tableau

F O X D R P B Z U L G E A T W N S M Y V C J H Q I K

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infinite word $\xrightarrow{\text{RSK}}$ recording tableau

F	O	X	

insertion tableau

1	2		

recording tableau

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infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\xrightarrow{\text{RSK}}$ recording tableau

F	O	X	

insertion tableau

1	2	3	

recording tableau

F O X D R P B Z U L G E A T W N S M Y V C J H Q I K

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infinite word $\xrightarrow{\text{RSK}}$ recording tableau

F	O	X	

insertion tableau

1	2	3	

recording tableau

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F	O	X	

insertion tableau

1	2	3	

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insertion tableau

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D	O	X	

insertion tableau

1	2	3	

recording tableau

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D	O	X	

insertion tableau

1	2	3	

recording tableau

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infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\xrightarrow{\text{RSK}}$ recording tableau

D	O	X	

insertion tableau

1	2	3	

recording tableau

F O X D R P B Z U L G E A T W N S M Y V C J H Q I K

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infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\xrightarrow{\text{RSK}}$ recording tableau

F			
D	O	X	

insertion tableau

1	2	3	

recording tableau

F O X **D** R P B Z U L G E A T W N S M Y V C J H Q I K

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infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\xrightarrow{\text{RSK}}$ recording tableau

F			
D	O	X	

insertion tableau

4			
1	2	3	

recording tableau

F O X **D** R P B Z U L G E A T W N S M Y V C J H Q I K

- ① *start from the first row,*
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infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\xrightarrow{\text{RSK}}$ recording tableau

F			
D	O	X	

insertion tableau

4			
1	2	3	

recording tableau

F O X **D** R P B Z U L G E A T W N S M Y V C J H Q I K

- ① *start from the first row,*
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- ④ *information about the new box into the recording tableau,*

infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\xrightarrow{\text{RSK}}$ recording tableau

F			
D	O	X	

insertion tableau

4			
1	2	3	

recording tableau

F O X D **R** P B Z U L G E A T W N S M Y V C J H Q I K

- ① *start from the first row,*
- ② *insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,*
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- ④ *information about the new box into the recording tableau,*

infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\xrightarrow{\text{RSK}}$ recording tableau

F			
D	O	X	

insertion tableau

4			
1	2	3	

recording tableau

F O X D R P B Z U L G E A T W N S M Y V C J H Q I K

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infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\xrightarrow{\text{RSK}}$ recording tableau

F			
D	O	X	

insertion tableau

4			
1	2	3	

recording tableau

F O X D R P B Z U L G E A T W N S M Y V C J H Q I K

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infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\xrightarrow{\text{RSK}}$ recording tableau

F			
D	O		

insertion tableau

4			
1	2	3	

recording tableau

F O X D **R** P B Z U L G E A T W N S M Y V C J H Q I K

- ① *start from the first row,*
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infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\xrightarrow{\text{RSK}}$ recording tableau

F			
D	O	R	

insertion tableau

4			
1	2	3	

recording tableau

F O X D R P B Z U L G E A T W N S M Y V C J H Q I K

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infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\xrightarrow{\text{RSK}}$ recording tableau

F			
D	O	R	

insertion tableau

4			
1	2	3	

recording tableau

F O X D R P B Z U L G E A T W N S M Y V C J H Q I K

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infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\xrightarrow{\text{RSK}}$ recording tableau

F			
D	O	R	

insertion tableau

4			
1	2	3	

recording tableau

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infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\xrightarrow{\text{RSK}}$ recording tableau

F	X		
D	O	R	

insertion tableau

4			
1	2	3	

recording tableau

F O X D R P B Z U L G E A T W N S M Y V C J H Q I K

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infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\xrightarrow{\text{RSK}}$ recording tableau

F	X		
D	O	R	

insertion tableau

4	5		
1	2	3	

recording tableau

F O X D R P B Z U L G E A T W N S M Y V C J H Q I K

- ① start from the first row,
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infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\xrightarrow{\text{RSK}}$ recording tableau

F	X		
D	O	R	

insertion tableau

4	5		
1	2	3	

recording tableau

F O X D **R** P B Z U L G E A T W N S M Y V C J H Q I K

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infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\xrightarrow{\text{RSK}}$ recording tableau

F	X		
D	O	R	

insertion tableau

4	5		
1	2	3	

recording tableau

F O X D R **P** B Z U L G E A T W N S M Y V C J H Q I K

- ① *start from the first row,*
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infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\xrightarrow{\text{RSK}}$ recording tableau

F	X		
D	O	R	

insertion tableau

4	5		
1	2	3	

recording tableau

F O X D R P B Z U L G E A T W N S M Y V C J H Q I K

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infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\xrightarrow{\text{RSK}}$ recording tableau

F	X		
D	O	R	

insertion tableau

4	5		
1	2	3	

recording tableau

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infinite word $\xrightarrow{\text{RSK}}$ recording tableau

F	X		
D	O		

insertion tableau

4	5		
1	2	3	

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F O X D R P B Z U L G E A T W N S M Y V C J H Q I K

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infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\xrightarrow{\text{RSK}}$ recording tableau

F	X		
D	O	P	

insertion tableau

4	5		
1	2	3	

recording tableau

F O X D R P B Z U L G E A T W N S M Y V C J H Q I K

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infinite Robinson-Schensted-Knuth (RSK) map

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F	X		
D	O	P	

insertion tableau

4	5		
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infinite word $\xrightarrow{\text{RSK}}$ recording tableau

F	X		
D	O	P	

insertion tableau

4	5		
1	2	3	

recording tableau

F O X D R P B Z U L G E A T W N S M Y V C J H Q I K

- ① start from the first row,
- ② insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,
- ③ insert the bumped element into the next row,
- ④ information about the new box into the recording tableau,

infinite Robinson-Schensted-Knuth (RSK) map

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infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\xrightarrow{\text{RSK}}$ recording tableau

X			
F	R		
D	O	P	

insertion tableau

6			
4	5		
1	2	3	

recording tableau

F O X D R P B Z U L G E A T W N S M Y V C J H Q I K

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infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\xrightarrow{\text{RSK}}$ recording tableau

X			
F	R		
D	O	P	

insertion tableau

6			
4	5		
1	2	3	

recording tableau

F O X D R **P** B Z U L G E A T W N S M Y V C J H Q I K

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infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\xrightarrow{\text{RSK}}$ recording tableau

F	L	N	T
D	G	M	S
B	E	J	Q
A	C	H	I

insertion tableau

7	16	22	25
6	10	14	24
4	5	9	17
1	2	3	8

recording tableau

F O X D R P B Z U L G E A T W N S M Y V C J H Q I K

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~~insertion tableau~~

recording tableau

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6	10	14	24
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1	2	3	8

~~insertion tableau~~

recording tableau

F O X D R P B Z U L G E A T W N S M Y V C J H Q I K

if X_0, X_1, \dots are i.i.d. $U(0, 1)$ random variables then

$\text{RSK}(X_0, X_1, \dots) \stackrel{\text{distribution}}{=} \text{Plancherel measure}$

8	13	18	32
6	9	12	23
4	5	7	19
1	2	3	10

Jeu de taquin

① start with $t \in \Omega$,

8	13	18	32
6	9	12	23
4	5	7	19
1	2	3	10

Jeu de taquin

- ① start with $t \in \Omega$,
- ② remove corner box,

8	13	18	32
6	9	12	23
4	5	7	19
	2	3	10

Jeu de taquin

- ① start with $t \in \Omega$,
- ② remove corner box,

8	13	18	32
6	9	12	23
4	5	7	19
	2	3	10

Jeu de taquin

- ① start with $t \in \Omega$,
- ② remove corner box,
- ③ sliding,

8	13	18	32
6	9	12	23
4	5	7	19
2		3	10

Jeu de taquin

- ① start with $t \in \Omega$,
- ② remove corner box,
- ③ sliding,

8	13	18	32
6	9	12	23
4	5	7	19
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- ① start with $t \in \Omega$,
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8	13	24	32
6	9	18	23
4	5	12	19
2	3	7	10

Jeu de taquin

- ① start with $t \in \Omega$,
- ② remove corner box,
- ③ sliding,
- ④ subtract 1 from all boxes

7	12	23	31
5	8	17	22
3	4	11	18
1	2	6	9

Jeu de taquin

- ① start with $t \in \Omega$,
- ② remove corner box,
- ③ sliding,
- ④ subtract 1 from all boxes

7	12	23	31
5	8	17	22
3	4	11	18
1	2	6	9

Jeu de taquin

- ① start with $t \in \Omega$,
- ② remove corner box,
- ③ sliding,
- ④ subtract 1 from all boxes

output:

- new tableau $J(t)$,
- trajectory $\mathbf{c}(t) = (c_1, c_2, \dots)$

7	12	23	31
5	8	17	22
3	4	11	18
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Jeu de taquin

- ① start with $t \in \Omega$,
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- ③ sliding,
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output:

- new tableau $J(t)$,
- trajectory $\mathbf{c}(t) = (c_1, c_2, \dots)$

'how representation of $S_{\{1,2,\dots\}}$ is related to its restriction to $S_{\{2,3,\dots\}}$ '?

Jeu de taquin - overview

8	13	18	32
6	9	12	23
4	5	7	19
1	2	3	10

original tableau t

8	13	24	32
6	9	18	23
4	5	12	19
2	3	7	10

outcome of slidings

7	12	23	31
5	8	17	22
3	4	11	18
1	2	6	9

new tableau $J(t)$

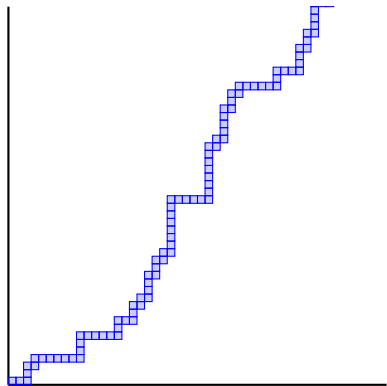
RSK

jeu de taquin

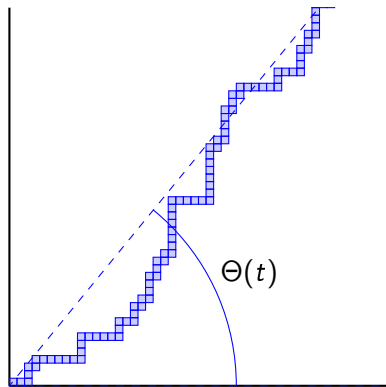
Littlewood-Richardson rule

$$V^\lambda \otimes V^\mu \uparrow_{S_n \times S_k}^{S_{n+k}} = \bigoplus_{\nu} c_{\lambda\mu}^{\nu} V^{\nu}$$

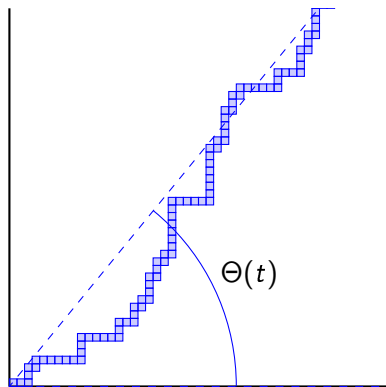
trajectories of jeu de taquin



trajectories of jeu de taquin



trajectories of jeu de taquin



if $t \in \Omega$ is random,
Plancherel distributed

then its jdt trajectory $\mathbf{c}(t)$
is almost surely asymptotically
a straight line,

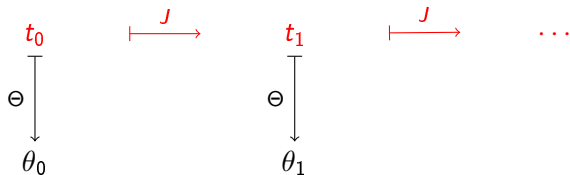
i.e.

$$\lim_{k \rightarrow \infty} \frac{c_k}{\|c_k\|} = (\cos \Theta(t), \sin \Theta(t))$$

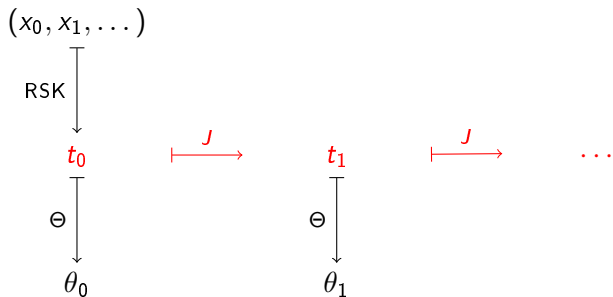
exists almost surely

$$\begin{array}{ccc} t_0 & \xrightarrow{J} & t_1 \\ \Theta \downarrow & & \\ \theta_0 & & \end{array}$$

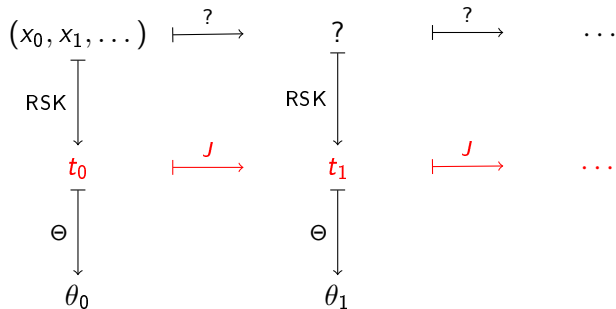
$$\begin{array}{ccc} t_0 & \xrightarrow{J} & t_1 & \xrightarrow{J} & \dots \\ \Theta \downarrow & & \Theta \downarrow & & \\ \theta_0 & & \theta_1 & & \end{array}$$



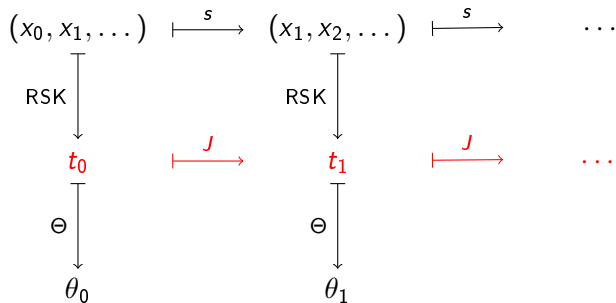
jeu de taquin dynamical system $(\Omega, \text{Plancherel}, J)$



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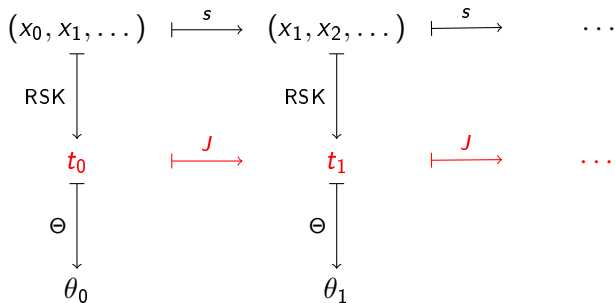


jeu de taquin dynamical system $(\Omega, \text{Plancherel}, J)$



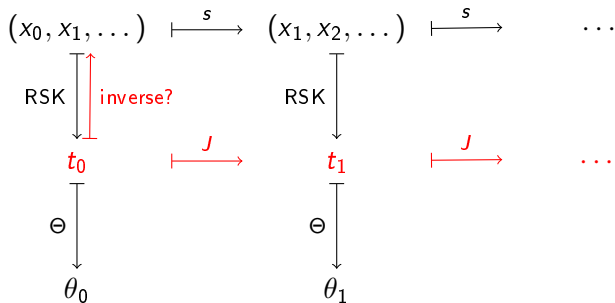
jeu de taquin dynamical system $(\Omega, \text{Plancherel}, J)$

i.i.d. shift dynamical system $([0, 1]^{\mathbb{N}}, \prod \text{Lebesgue}, s)$



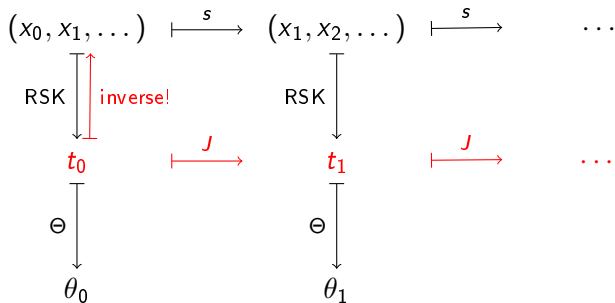
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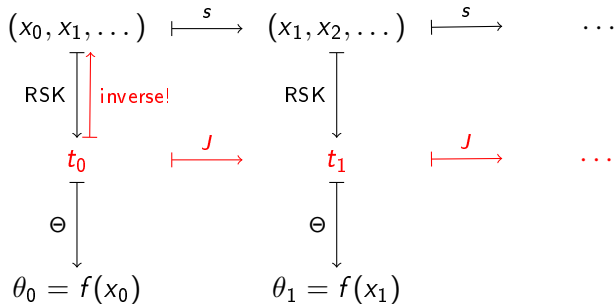
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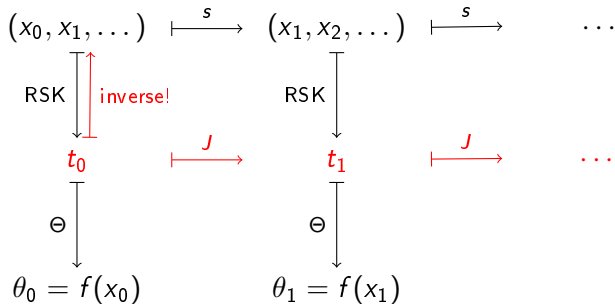
jeu de taquin dynamical system $(\Omega, \text{Plancherel}, J)$

i.i.d. shift dynamical system $([0, 1]^{\mathbb{N}}, \prod \text{Lebesgue}, s)$



jeu de taquin dynamical system $(\Omega, \text{Plancherel}, J)$

i.i.d. shift dynamical system $([0, 1]^{\mathbb{N}}, \prod \text{Lebesgue}, s)$



jeu de taquin dynamical system $(\Omega, \text{Plancherel}, J)$

the jeu de taquin dynamical system is isomorphic to i.i.d. shift

the inverse map is given by $x_i = f^{-1}(\theta_i)$

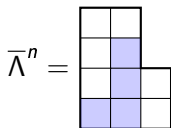
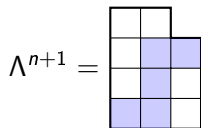
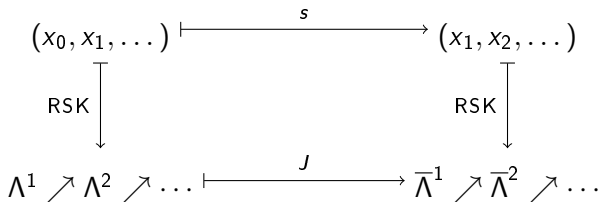
some consequences of the isomorphism:

- jdt is a measure-preserving transformation,
- jdt is ergodic,
- slope angles $\theta_0, \theta_1, \dots$ are independent random variables (put paths $\mathbf{c}(t_0), \mathbf{c}(t_1), \dots$ are not independent),
- generalizations to other probability measures on Ω / other representations of S_∞ ,

Why Θ exists and is a function of x_0 ?

x_0 is fixed

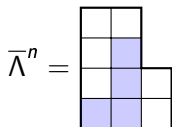
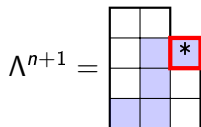
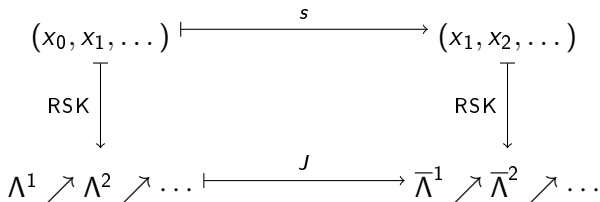
x_1, x_2, \dots are random, i.i.d. $U(0, 1)$



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x_0 is fixed

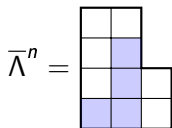
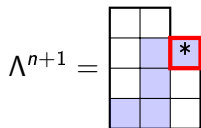
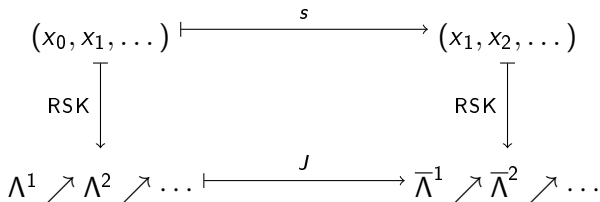
x_1, x_2, \dots are random, i.i.d. $U(0, 1)$



Why Θ exists and is a function of x_0 ?

x_0 is fixed

x_1, x_2, \dots are random, i.i.d. $U(0, 1)$



$$* = \Lambda^{n+1} \setminus \bar{\Lambda}^n = \text{RSK}(x_0, \dots, x_n) \setminus \text{RSK}(x_1, \dots, x_n)$$

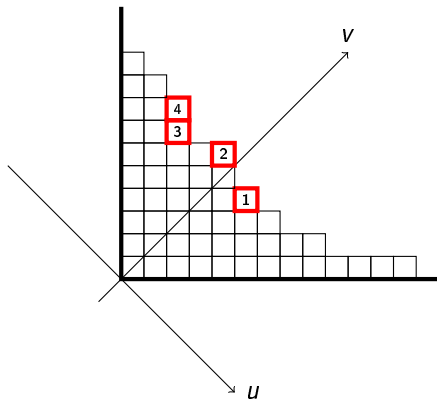
Is it true that asymptotically position of $*$ depends only on x_0 ?

instead of (for deterministic x_0) studying

$$\text{RSK}(x_0, \dots, x_n) \setminus \text{RSK}(x_1, \dots, x_n) = \boxed{*}$$

we study (for random $0 < t_1 < \dots < t_k < 1$)

$$\text{RSK}(t_1, \dots, t_k, x_1, \dots, x_n) \setminus \text{RSK}(x_1, \dots, x_n) = \{\boxed{1}, \dots, \boxed{k}\}$$



Plactic Littlewood-Richarson rule

if $0 \leq x_1, \dots, x_n \leq 1$ is a random sequence, conditioned in such a way that

$$\text{RSK}(x_1, \dots, x_n) = \lambda;$$

and $0 \leq t_1, \dots, t_k \leq 1$ is a random sequence, conditioned in such a way that

$$\text{RSK}(t_1, \dots, t_k) = \mu;$$

then the random Young diagram

$$\text{RSK}(t_1, \dots, t_k, x_1, \dots, x_n)$$

has the same distribution as random irreducible component of

$$V^\lambda \otimes V^\mu \uparrow_{S_n \times S_k}^{S_{n+k}}$$

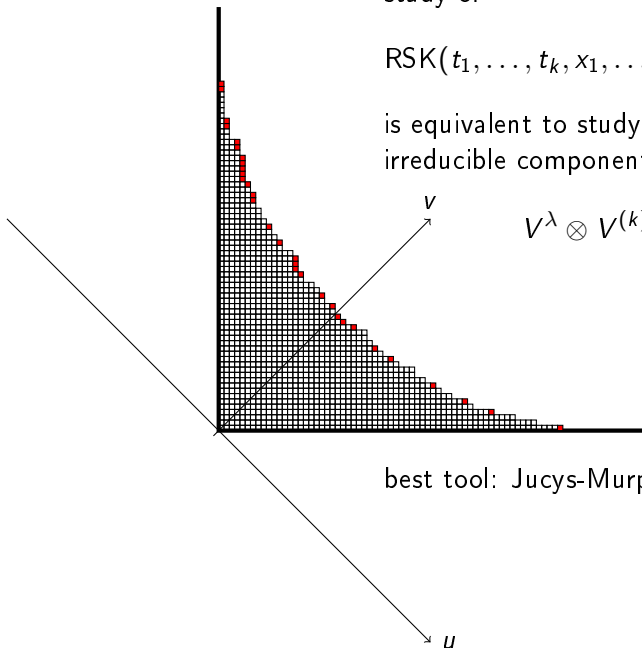
study of

$\text{RSK}(t_1, \dots, t_k, x_1, \dots, x_n) \setminus \text{RSK}(x_1, \dots, x_n)$

is equivalent to study of random
irreducible components of

$$V^\lambda \otimes V^{(k)} \uparrow_{S_n \times S_k}^{S_{n+k}}$$

best tool: Jucys-Murphy elements



Outlook: fluctuations?

